Analog Electronic

ENEE236

BJT AC Analysis

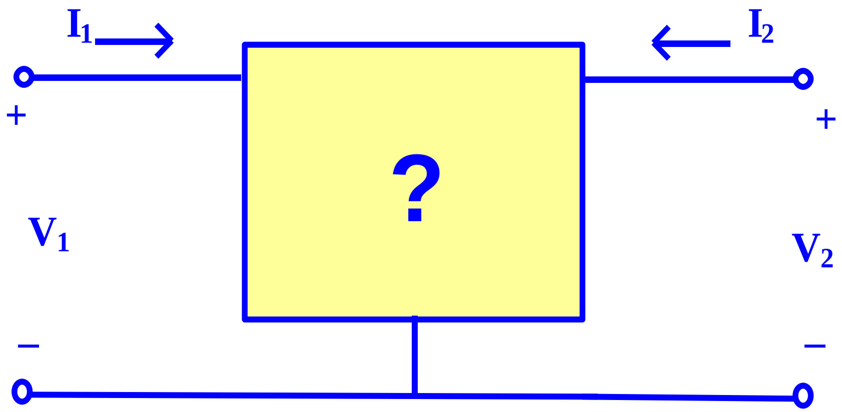
Chapter 5

Small Signal ac Equivalent Circuit

- In order to simplify the analysis, we replace the Transistor by an equivalent circuit (model)
- An AC model represents the AC characteristics of the transistor.
- A model uses circuit elements that approximate the behavior of the transistor.
- There are two models commonly used in small signal AC analysis of a transistor:
    - **r_e model**
    - **Hybrid equivalent model**

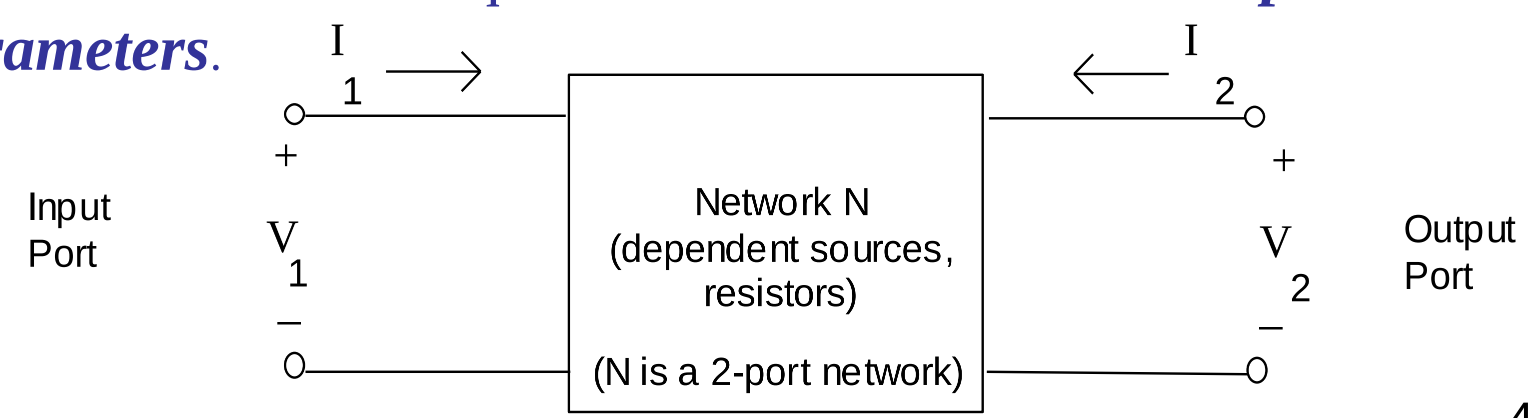
Modeling Two-Port Networks

- Two-port parameters can be determined for a given network.
- Additionally, two-port parameters might be specified for a certain device by the manufacturer (such as h-parameter values for a transistor).
- How are these parameters used?
- They are used to form a circuit model for the device or circuit. A circuit model is developed using the two-port parameter equations.

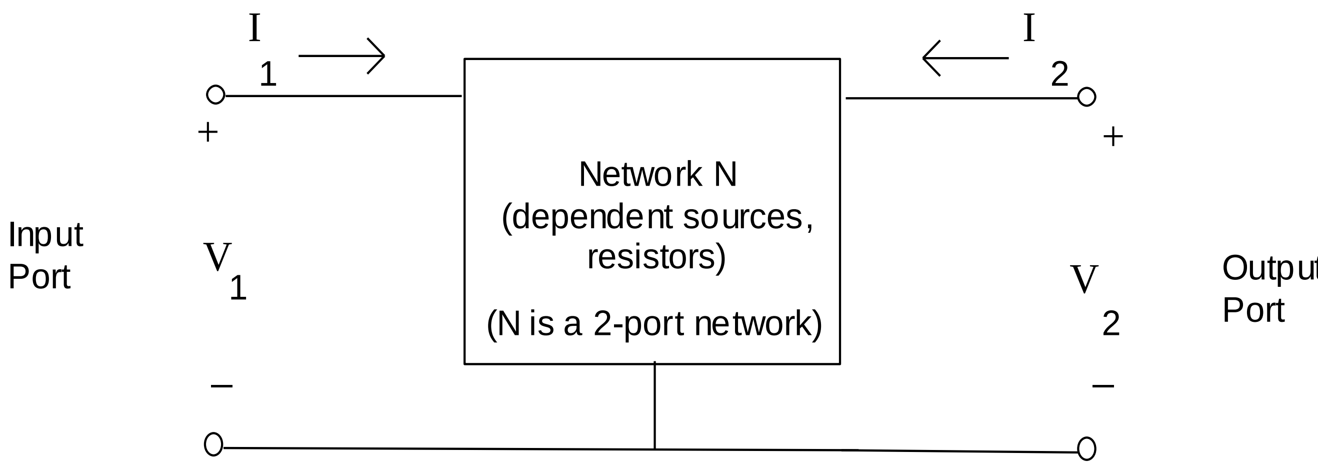


Two-port networks

- Suppose that a network N has two ports as shown below. How could it be represented or modeled?
- A common way to represent such a network is to use one of 6 possible ***two-port networks***.
- These networks are circuits that are based on one of 6 possible sets of ***two-port equations***. These equations are simply different combinations of two equations that relate the variables V_1, V_2, I_1, and I_2 to one another. The coefficients in these equations are referred to as ***two-port parameters***.



Note that I_1 , I_2 , V_1 , and V_2 are labeled as shown by convention. Often there is a common negative terminal between the input and the output so the figure above could be redrawn as:



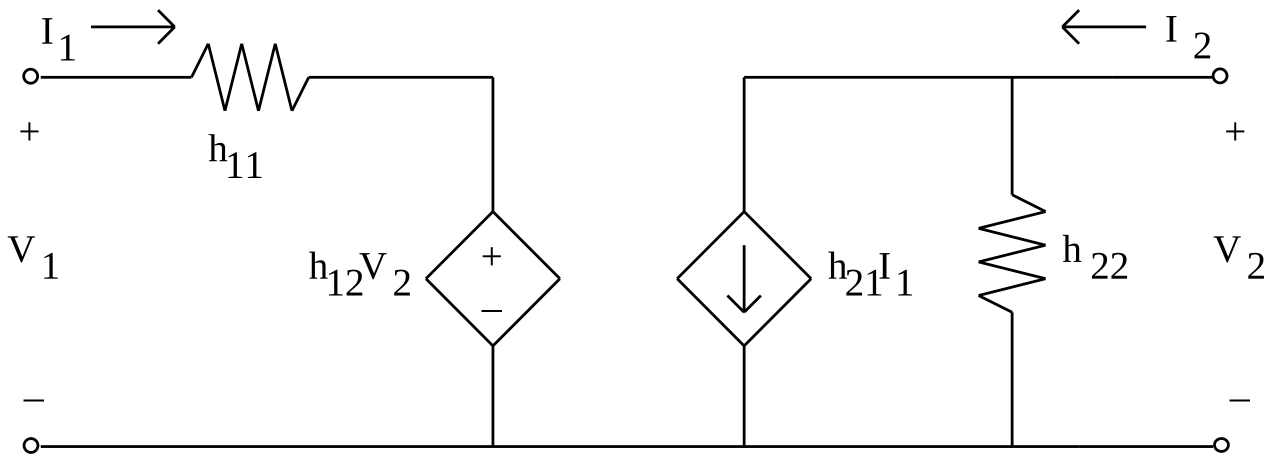
Development of the h-parameter model:

One possible circuit model could be developed by treating one of the two-port parameter equations as a KVL equation and the other as a KCL equation (illustrate). This results in the following circuit.

h - parameter equations:

$$V_1 = h_{11} \cdot I_1 + h_{12} \cdot V_2$$

$$I_2 = h_{21} \cdot I_1 + h_{22} \cdot V_2$$



$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

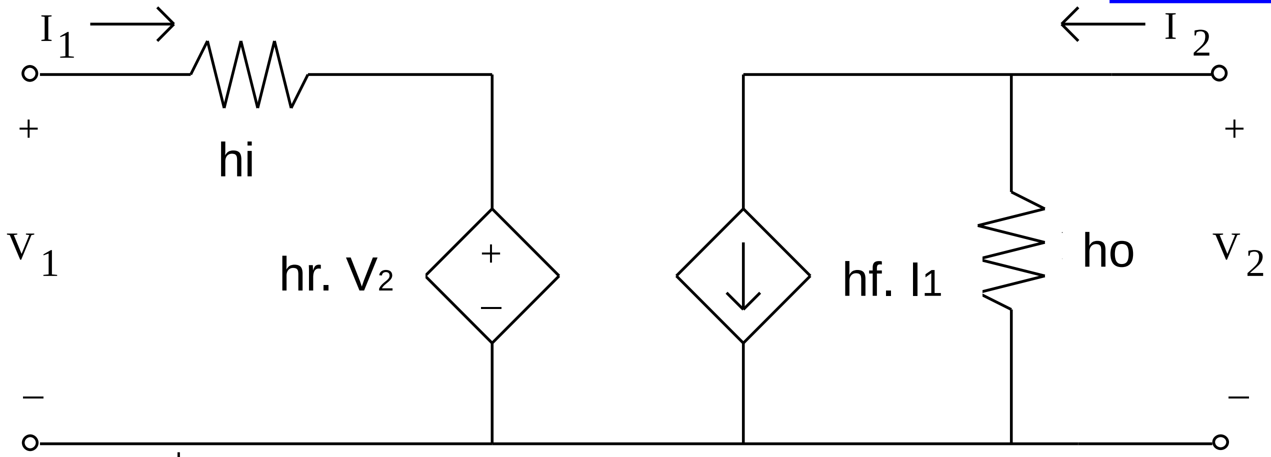
Development of the h-parameter model of BJT:

For A BJT the equivalent h parameter model can be described by the following equations:

h - parameter equations :

$$V_1 = h_i \cdot I_1 + h_r \cdot V_2$$

$$I_2 = h_f \cdot I_1 + h_o \cdot V_2$$

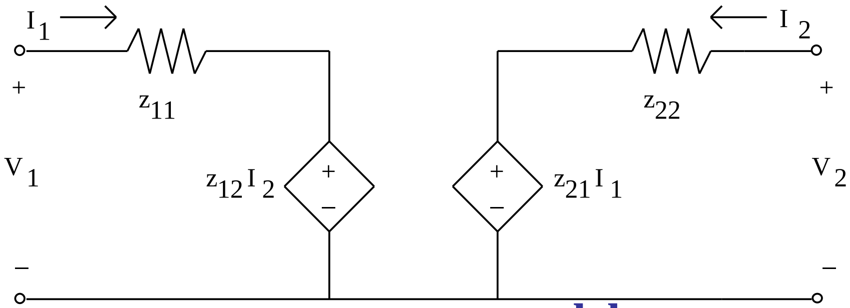


$$h_i = \left.\frac{V_1}{I_1}\right|_{V_2=0} \qquad h_r = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$h_f = \left.\frac{I_2}{I_1}\right|_{V_2=0} \qquad h_o = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

Summary:

Note: This page is for information only

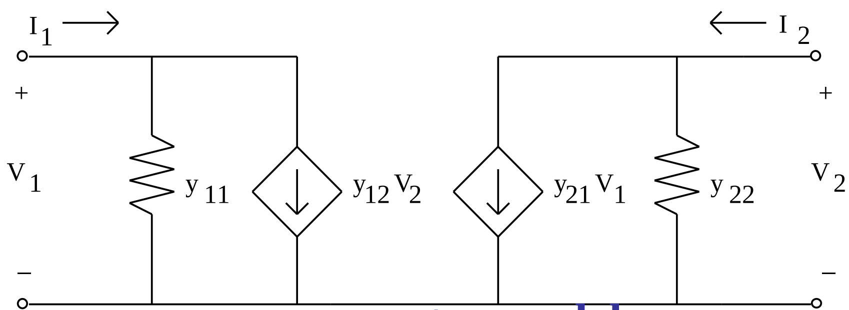


z-parameter model

z - parameter equations :

$$V_1 = z_{11}\cdot I_1 + z_{12}\cdot I_2$$

$$V_2 = z_{21}\cdot I_1 + z_{22}\cdot I_2$$

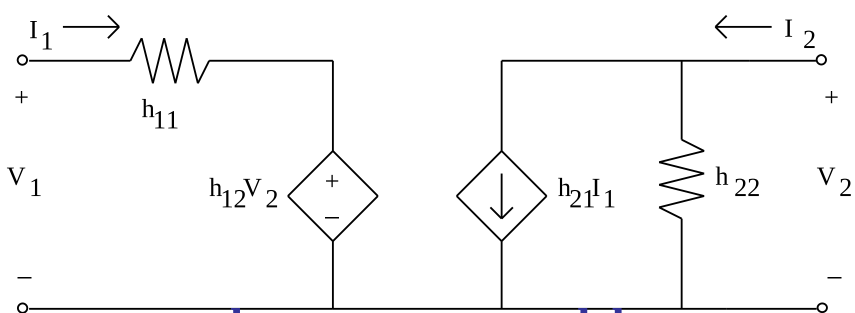


y-parameter model

y - parameter equations :

$$I_1 = y_{11}\cdot V_1 + y_{12}\cdot V_2$$

$$I_2 = y_{21}\cdot V_1 + y_{22}\cdot V_2$$



h-parameter model

h - parameter equations :

$$V_1 = h_{11}\cdot I_1 + h_{12}\cdot V_2$$

$$I_2 = h_{21}\cdot I_1 + h_{22}\cdot V_2$$

Two sets of measurements are made on a two-port resistive circuit. The first set is made with port 2 open, and the second set is made with port 2 short-circuited. The results are as follows:

Port 2 Open	Port 2 Short-Circuited
$V_1 = 10\text{ mV}$	$V_1 = 24\text{ mV}$
$I_1 = 10\ \mu\text{A}$	$I_1 = 20\ \mu\text{A}$
$V_2 = -40\text{ V}$	$I_2 = 1\text{ mA}$

Find the h parameters of the circuit.

h - parameter equations :

$$V_1 = h_{11} \cdot I_1 + h_{12} \cdot V_2$$

$$I_2 = h_{21} \cdot I_1 + h_{22} \cdot V_2$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \Omega, \qquad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0},$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}, \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} \text{S}.$$

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

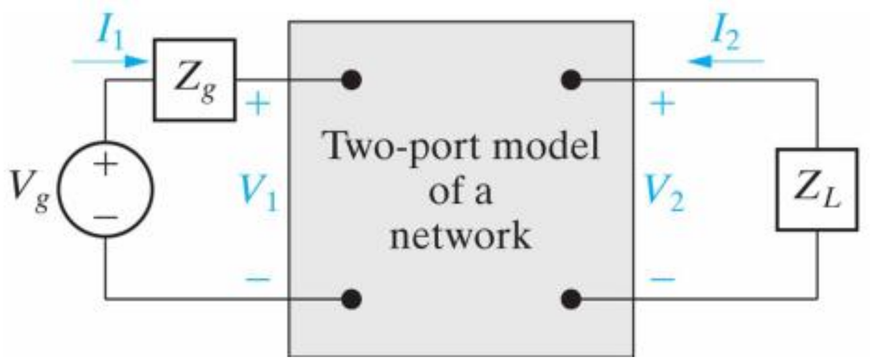
$$= \frac{24 \times 10^{-3}}{20 \times 10^{-6}} = 1.2\text{ k}\Omega,$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$= \frac{10^{-3}}{20 \times 10^{-6}} = 50.$$

BJT Configurations

- Common Emitter
- Common Base
- Common Collector



Terminated Two port network
Includes source and load

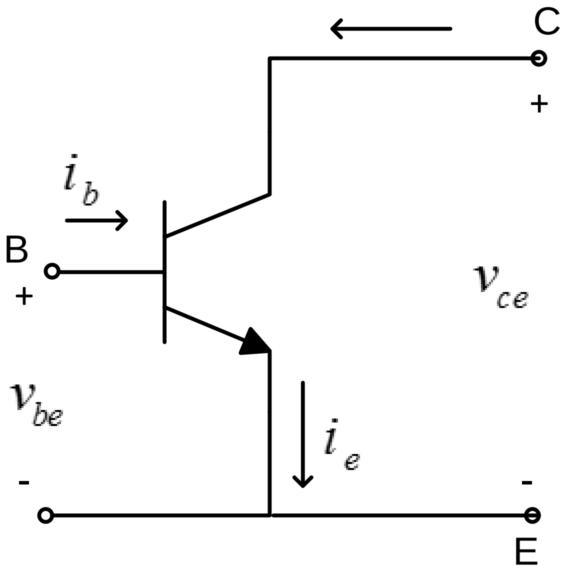
Common Emitter Configuration

(inverting configuration, provides voltage and current gain)

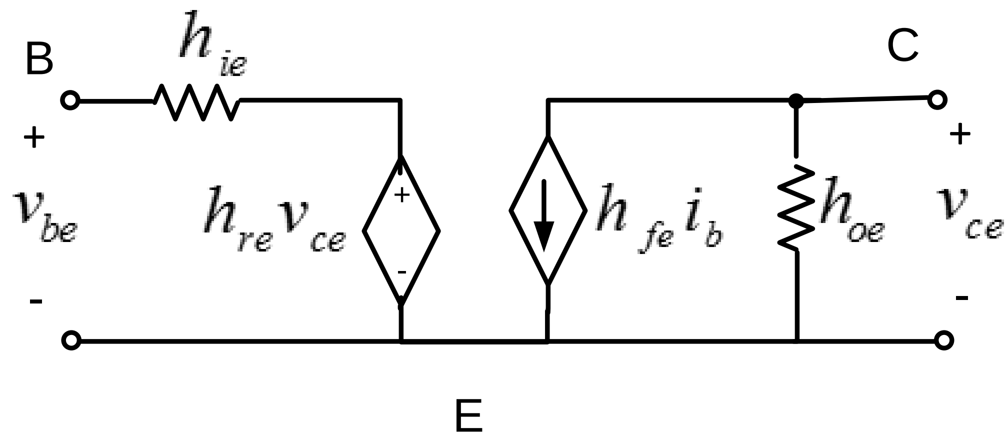
h - parameter equations:

$$V_{be} = h_{ie} \cdot I_b + h_{re} \cdot V_{ce}$$

$$I_c = h_{fe} \cdot I_b + h_{oe} \cdot V_{ce}$$



Detailed Model



Typical Data sheet parameter values

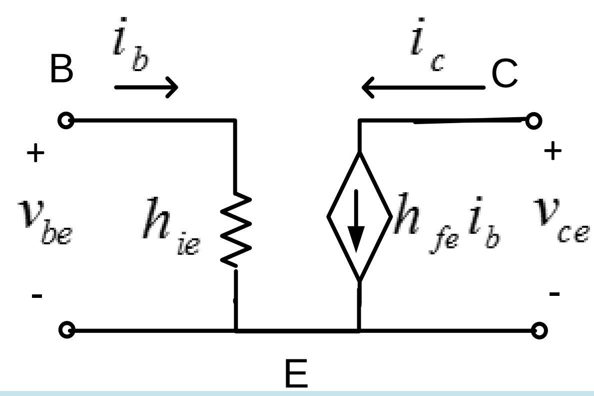
$h_{ie} \approx 1600\ \Omega$

$h_{re} \approx 0.0002$

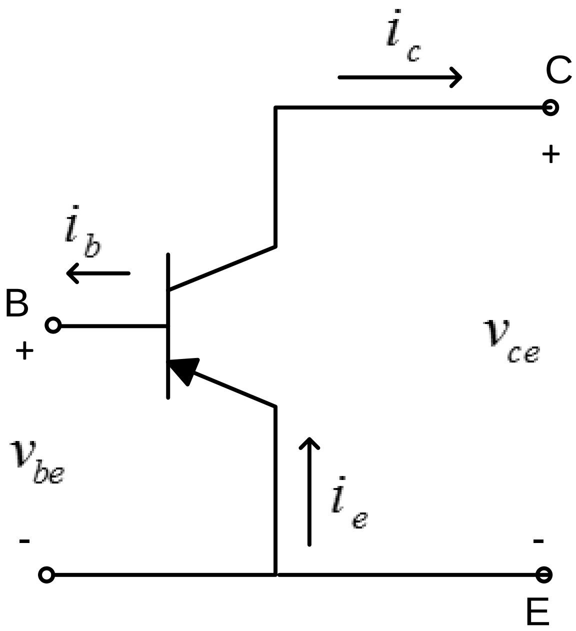
$h_{fe} \approx 80$

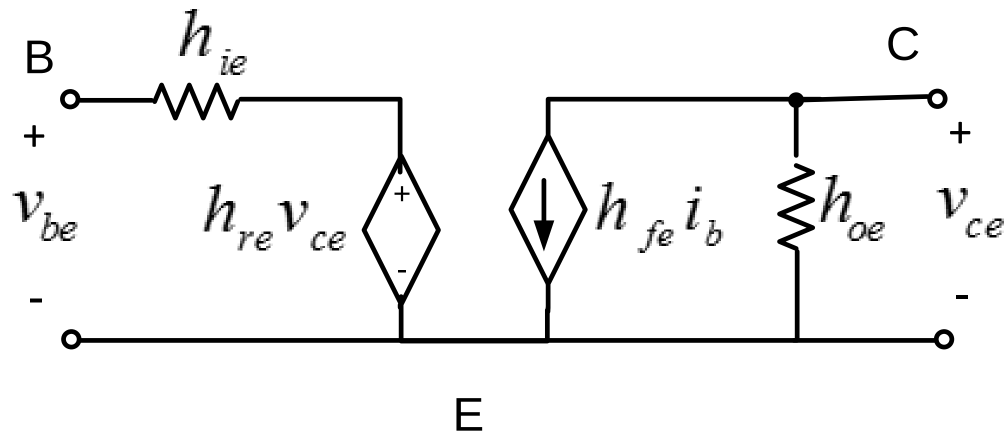
$h_{oe} \approx 20.10^{-6}$ Siemens

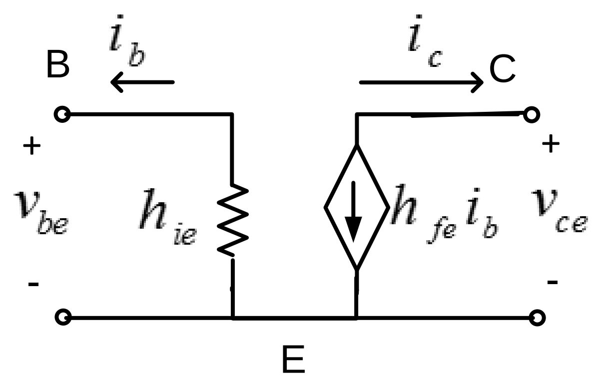
Simplified Model



Common Emitter and Common Collector Configuration







Value of hie

Base Emitter is a pn junction similar to a diode
hie is the dynamic resistance of the pn junction

In a diode:

$$r_d = \frac{V_T}{I_{DQ}} \Rightarrow$$

$$h_{ie} = \frac{V_T}{I_{BQ}} = \frac{V_T}{\frac{I_{CQ}}{h_{fe}}} = \frac{h_{fe} V_T}{I_{CQ}}$$

I_{BQ} dc value of base current

I_{CQ} dc value of collector current

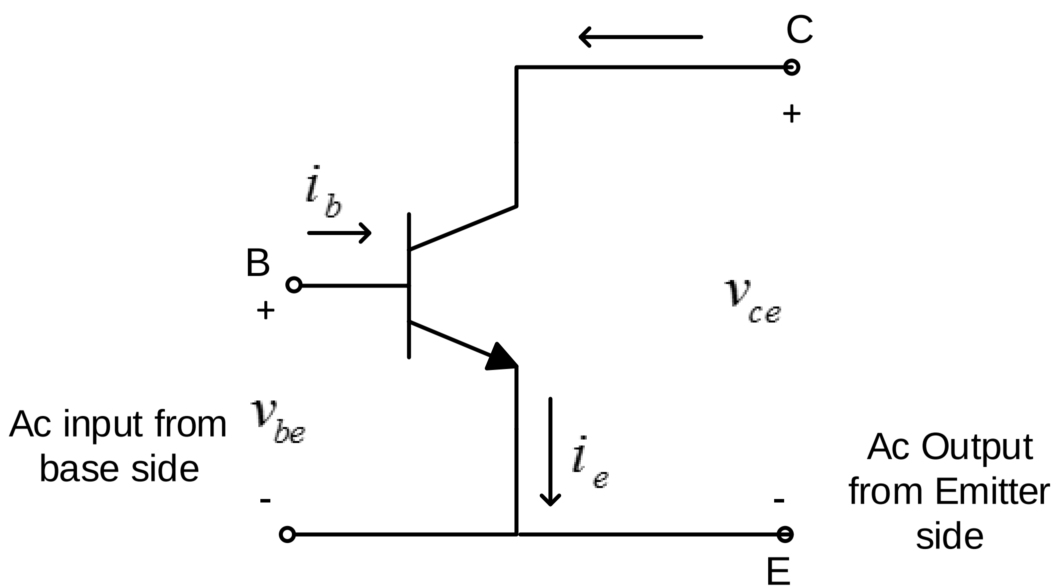
$$h_{fe} = \beta$$

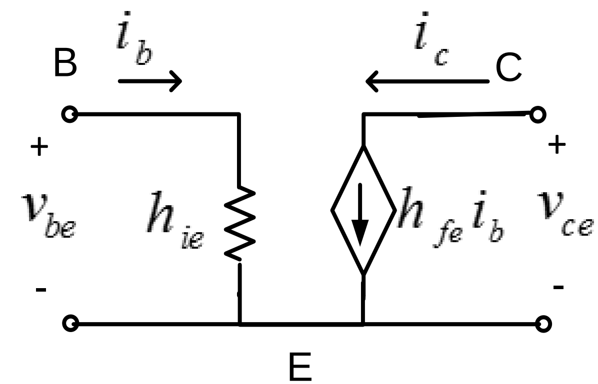
$$V_T = 25.69 \text{ mV } @ 25\ {}^{o}\text{C}$$

Common Collector

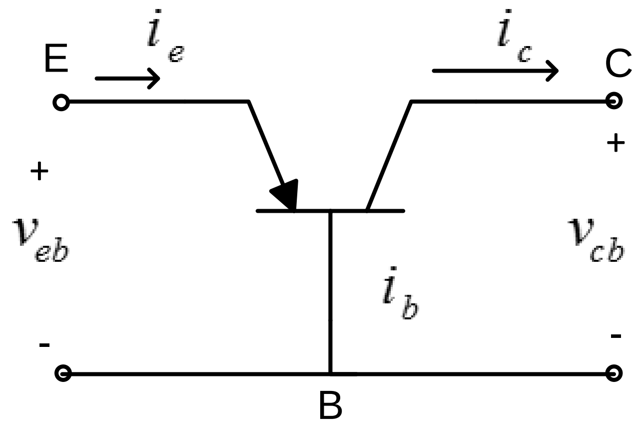
provides current gain and no voltage gain)

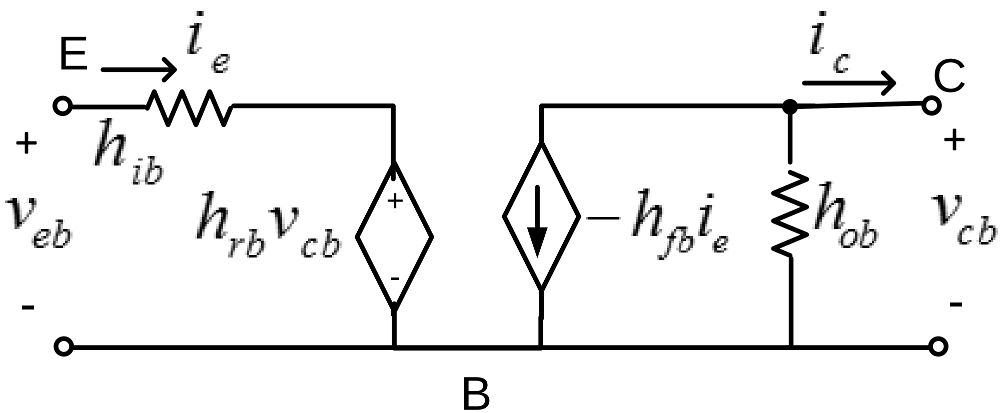
Same Model of Common Emitter will be used due to the similarities between them and for simplicity





Common-Base Configuration





h - parameter equations:

$$V_{eb} = h_{ib} \cdot I_e + h_{rb} \cdot V_{cb}$$

$$I_c = h_{fb} \cdot I_e + h_{ob} \cdot V_{cb}$$

$$h_{ib} = \left.\frac{V_{EB}}{I_E}\right|_{V_{CB}=0}$$

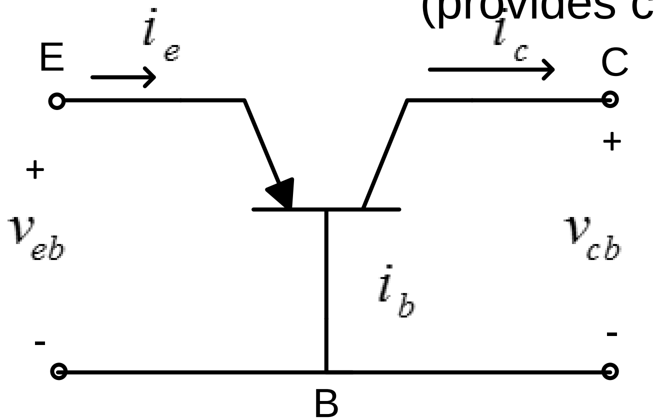
$$h_{rb} = \left.\frac{V_{EB}}{V_{CB}}\right|_{I_E=0}$$

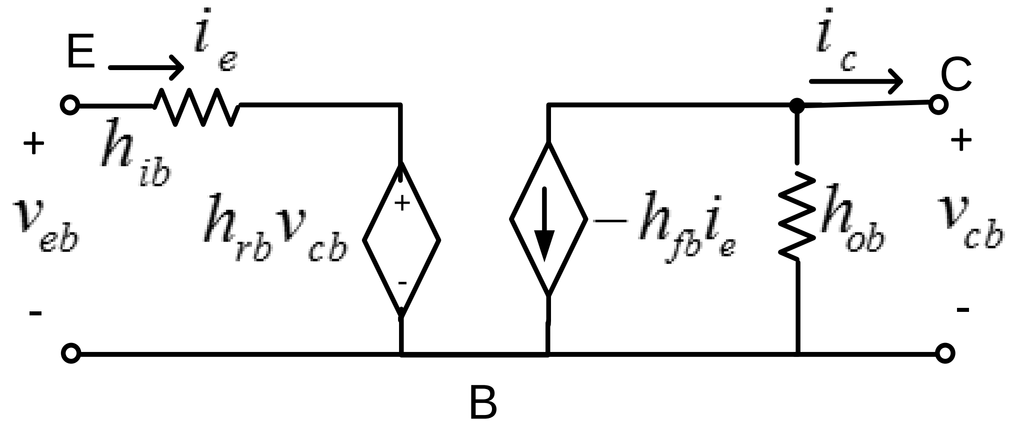
$$h_{fb} = \alpha = \left.\frac{I_C}{I_E}\right|_{V_{CB}=0}$$

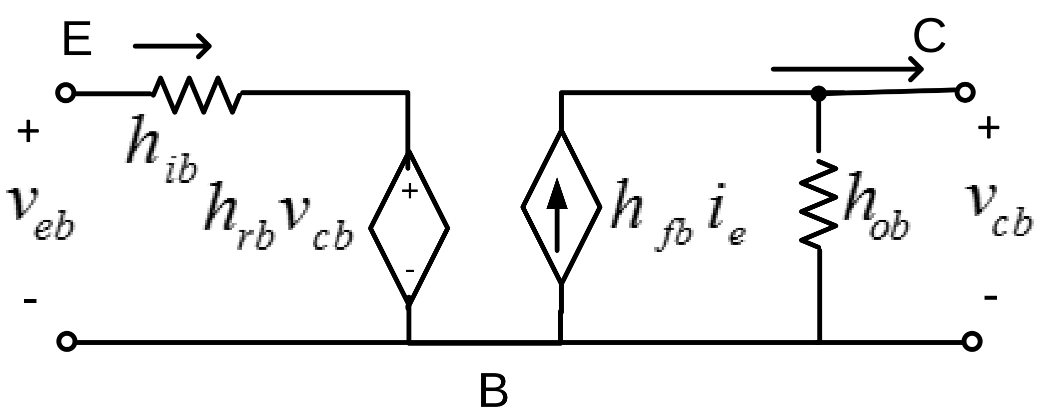
$$h_{ob} = \left.\frac{I_C}{V_{CB}}\right|_{I_E=0}$$

Common-Base Configuration

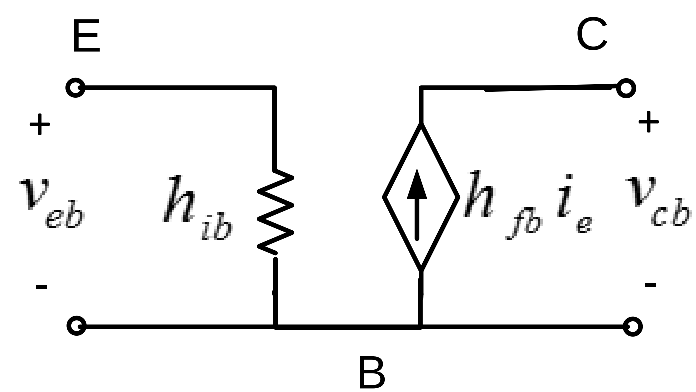
(provides current gain and some voltage gain)







Simplified Equivalent Circuit



Common-Base Configuration

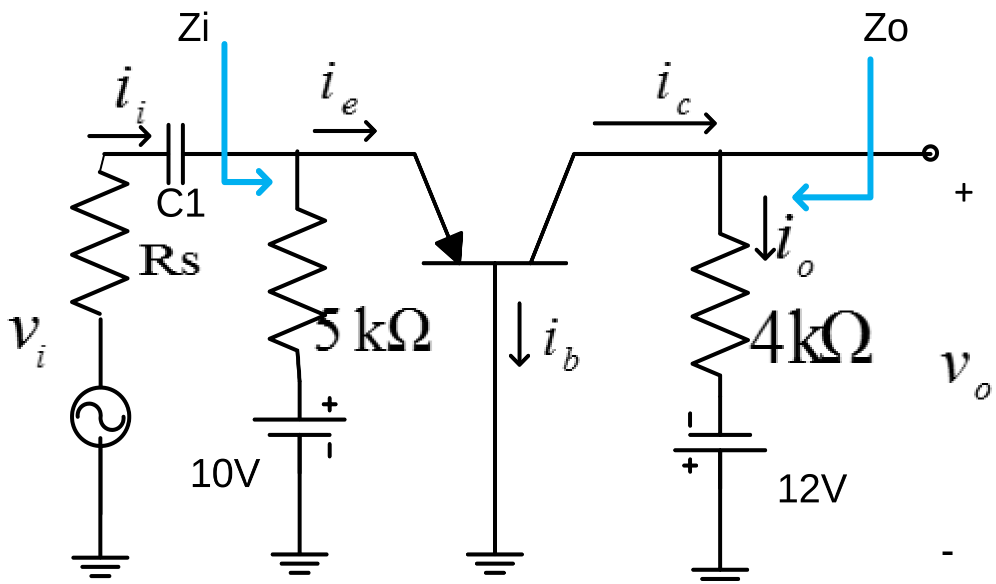
$$h_{ib} = \frac{V_T}{I_{EQ}}$$

$$h_{fb} = \alpha$$

$$V_T = 25.69 \ \text{mV} \ \ @\, 25\ ^{o}\text{C}$$

$$h_{ie} > h_{ib}$$

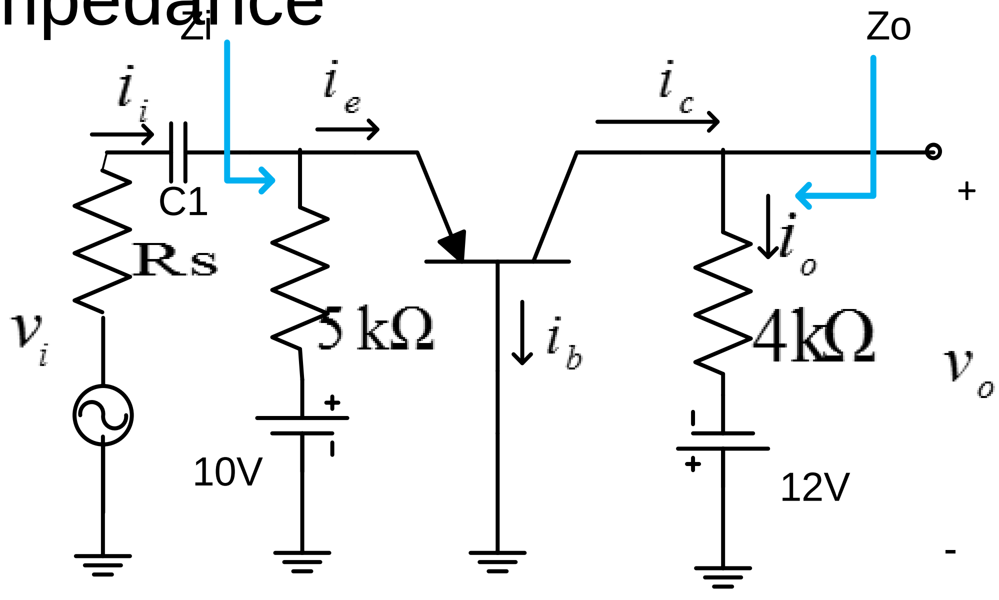
BJT Amplifier Analysis Example



BJT Amplifier Analysis

When Analyzing Amplifier Circuits, we usually want to find some or all of the following quantities:

1) Av=Vo/Vi, small signal voltage gain
2) Ai=io/ii,   small signal current gain
3) Zi   Input Impedance
4) Zo   Output Impedance



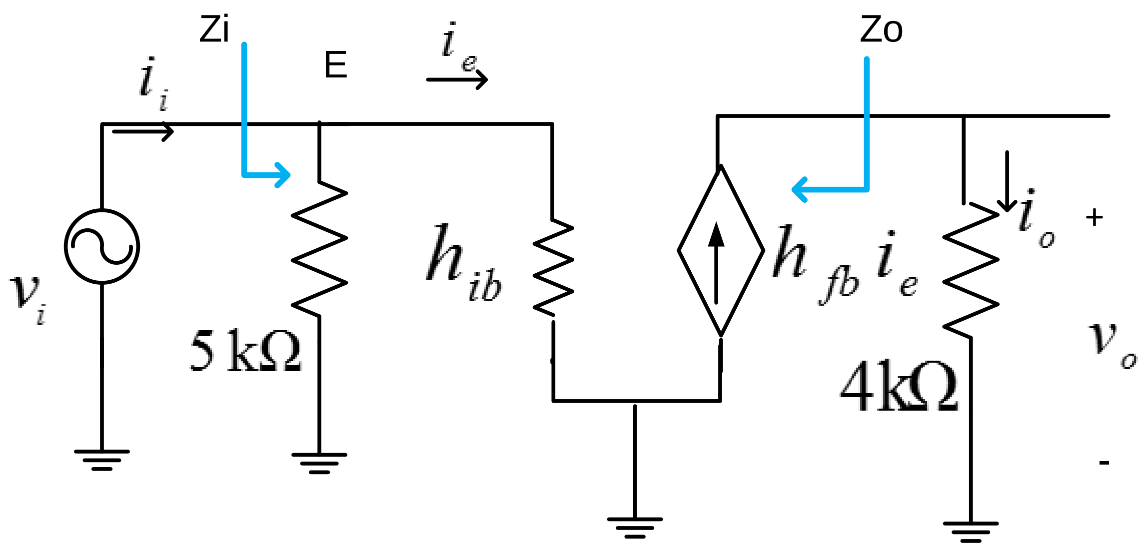
BJT Amplifier Analysis

Solution: (with Rs=0)

We draw the ac small signal equivalent circuit

Capacitors ==> replaced by short circuit

DC sources are killed ,



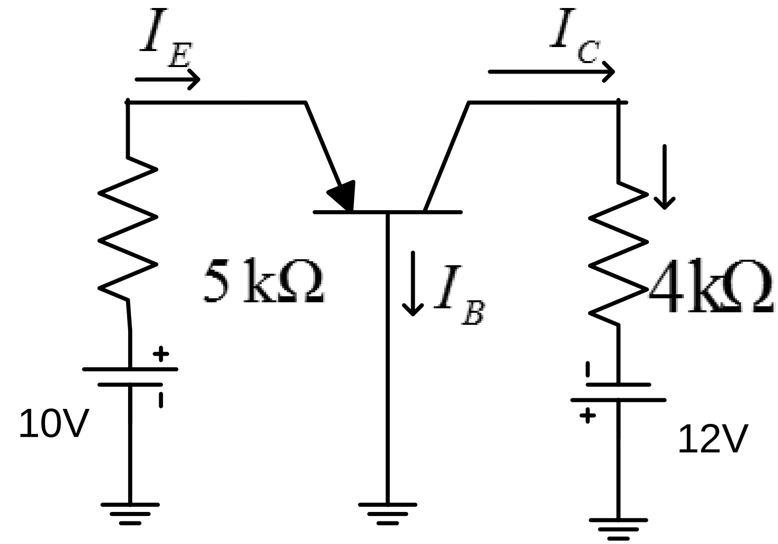
$$h_{ib} = \frac{V_T}{I_{EQ}}$$

$$h_{fb} = \alpha \cong 1$$

I_{EQ} must be calculated from DC analysis

DC Analysis

DC Equivalent Circuit:
-Cap ==> open
-Kill ac sources ==>



$$10 = 5\,k\Omega\,.I_{EQ} + V_{EB}$$

$$I_{EQ} = \frac{10 - 0.7}{5\,k\Omega} = 1.86\,mA$$

$$h_{ib} = \frac{V_T}{I_{EQ}} = \frac{25.69\,mV}{1.86\,mA} = 13.98\,\Omega$$

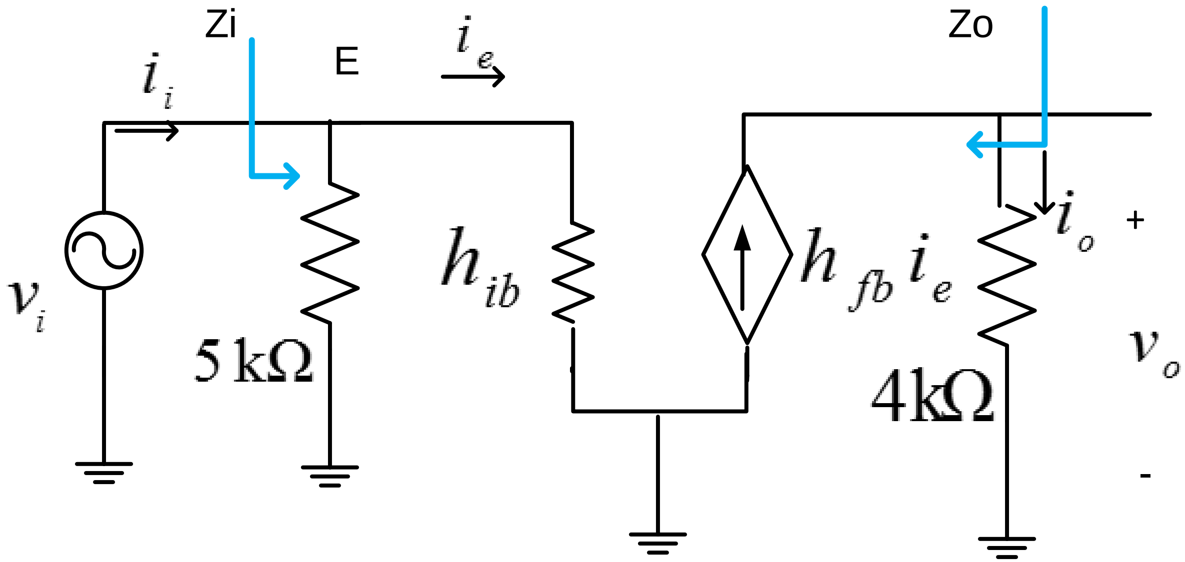
Ac ss equivalent circuit

$$1)\ A_V = \frac{v_o}{v_i}$$

$$v_o = i_o \cdot 4\,\mathrm{k\Omega}$$

$$i_o = h_{fb} \cdot i_e$$

$$i_e = \frac{v_i}{h_{ib}}$$



$$A_V = \frac{v_o}{v_i} = \frac{v_o}{i_o} \cdot \frac{i_o}{i_e} \cdot \frac{i_e}{v_i}$$

$$A_V = (4\,\mathrm{k\Omega}) \cdot (h_{fb}) \cdot \left(\frac{1}{h_{ib}}\right)$$

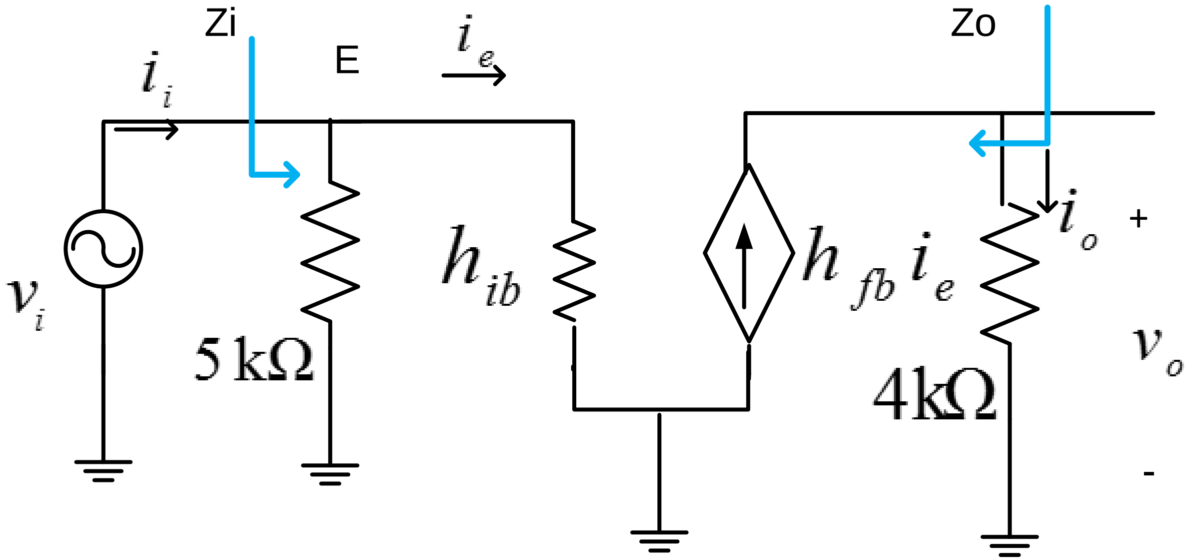
$$= (4\,\mathrm{k\Omega}) \cdot (1) \cdot \left(\frac{1}{13.98}\right) = 286 > 1$$

Current Gain Ai

2) $A_i = \dfrac{i_o}{i_i}$

$i_o = h_{fb}.i_e$

$i_e = i_i \dfrac{5\,k\Omega}{5\,k\Omega + h_{ib}}$

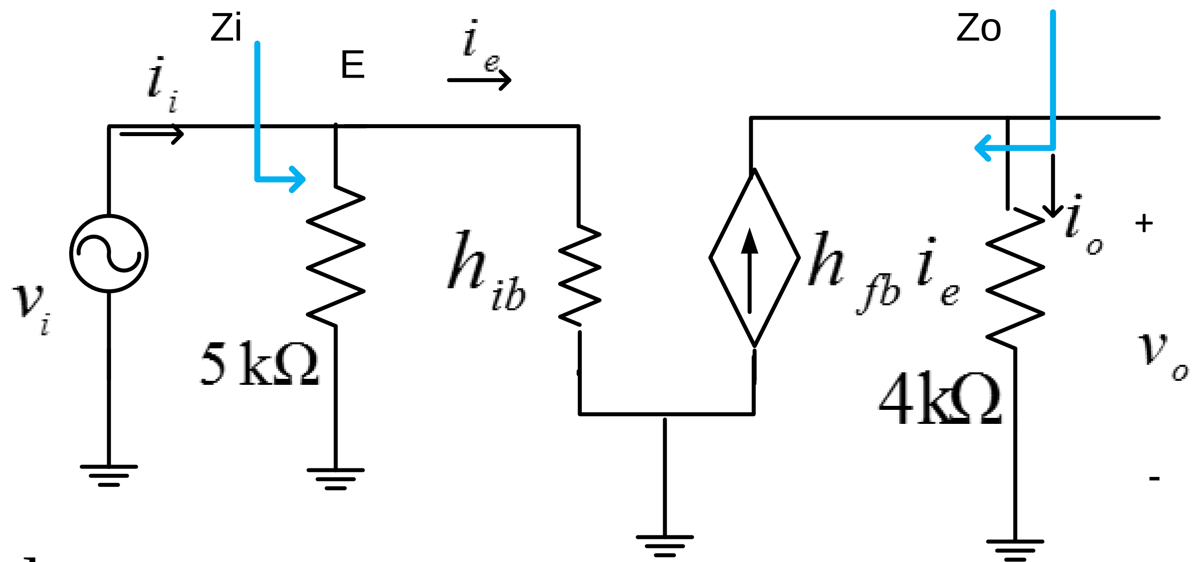


$$\Rightarrow A_i = \frac{i_o}{i_i} = \frac{i_o}{i_e}.\frac{i_e}{i_i}$$

$$\Rightarrow A_i = (h_{fb})\left(\frac{5\,k\Omega}{5\,k\Omega + h_{ib}}\right)$$

$$= (1)\left(\frac{5\,k\Omega}{5\,k\Omega + 13.98}\right) < 1$$

Zi & Zo



3) Input Impedance

$$Z_{\mathrm{i}} = \left(h_{ib} \,//\, 5\,\mathrm{k}\Omega\right) = \left(\frac{h_{ib} . 5\,\mathrm{k}\Omega}{5\,\mathrm{k}\Omega + h_{ib}}\right)$$

4) Output Impedance

$$Z_o\Big|_{\text{all independant sources killed (i.e. Vi=0 or short)}} = 4\,\mathrm{k}\Omega$$

With Presence of Rs

with R_S

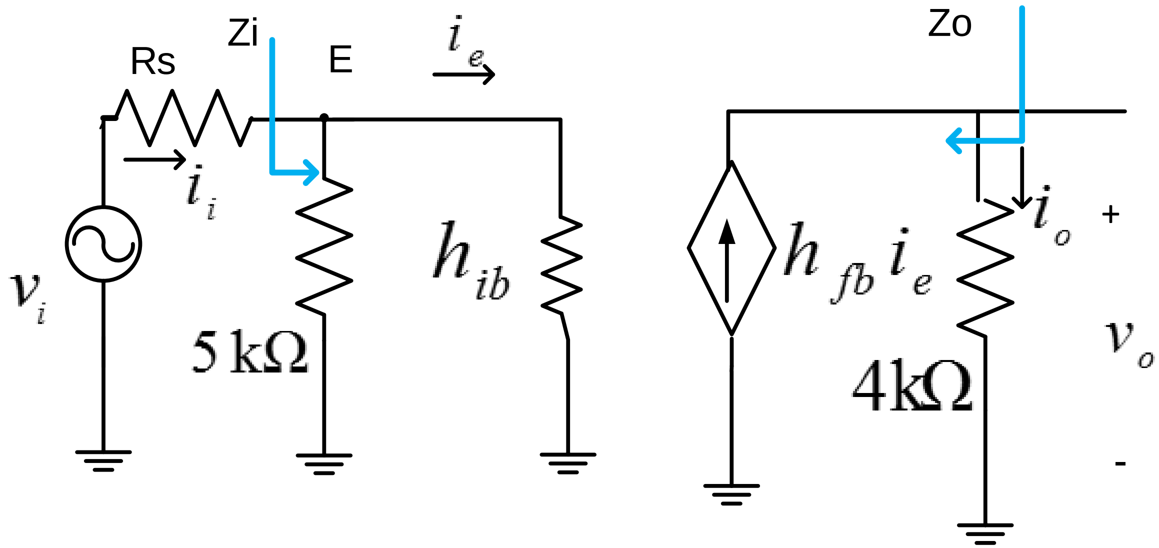
$$i_i = \frac{v_i}{Z_i + R_s}$$

For $Rs = 50\,\Omega$

$A_V = 62.5$

For $Rs = 10\,k\Omega$

$A_V = 0.4$

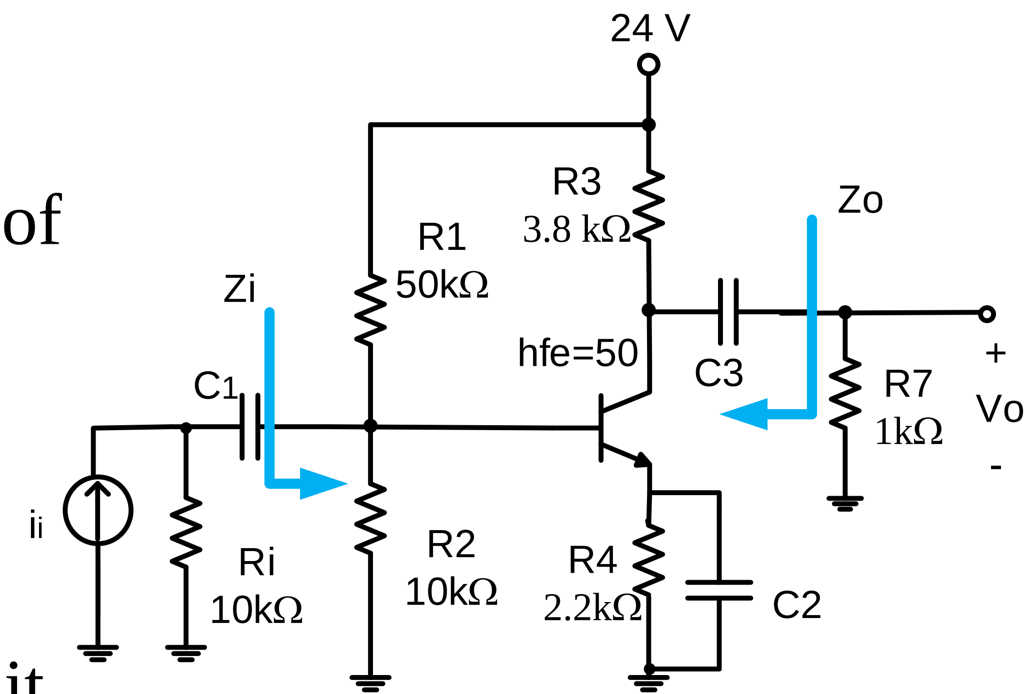


Example: Common Emitter (CE)

1) From DC Analysis,

we find Q - point and value of

$$h_{ie} = \frac{V_T}{I_{BQ}}$$

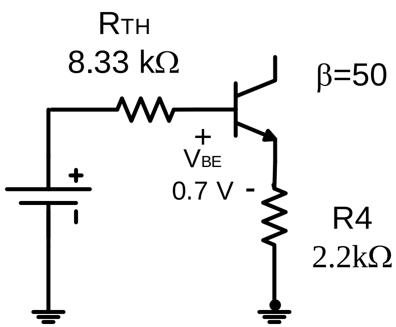


Thevenin's equivalent circuit

as seen from the base

$$V_{TH} = \frac{10\ \text{k}\Omega}{10\ \text{k}\Omega + 50\ \text{k}\Omega} .24\ \text{V} = 4\ \text{V}$$

$$\text{R}_{\text{TH}} = 10\ \text{k}\Omega\Omega // 5\text{k}\Omega = 8.33\ \text{k}\Omega$$

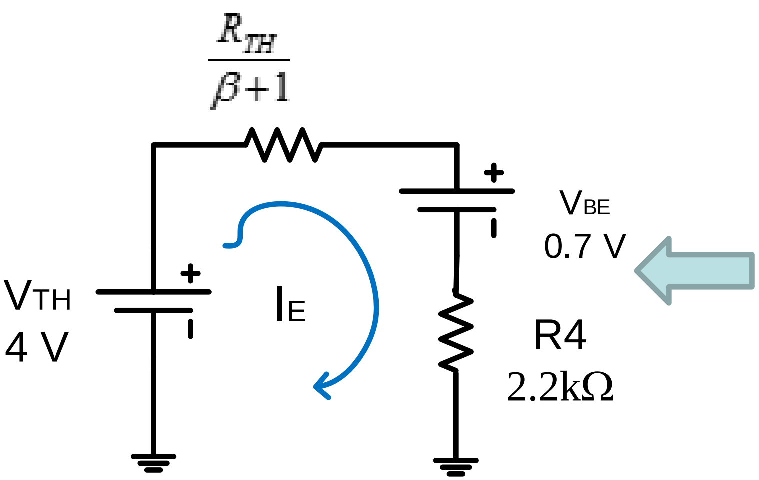


$$4 = 8.33\,\mathrm{k\Omega}\,.\,I_B + V_{BE} + 2.2\,\mathrm{k\Omega}.\,I_E$$

But, $\quad I_E = (1+\beta)I_B$

$$\text{Solve for } I_E = \frac{4 - 0.7}{\dfrac{8.33\,\mathrm{k\Omega}}{(1+50)} + 2.2\,\mathrm{k\Omega}} = 1.4\,\mathrm{mA}$$

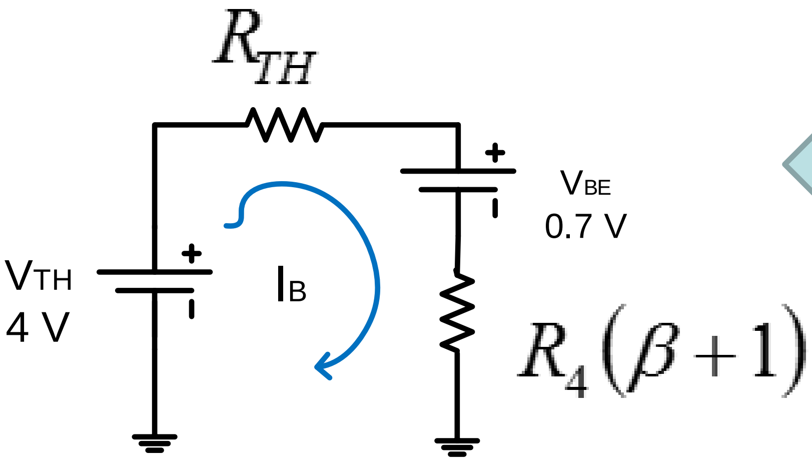
$$h_{ie} = \frac{V_T}{I_{BQ}} = \frac{25.69\,\mathrm{mV}}{\dfrac{1.4\,\mathrm{mA}}{51}} = 928\,\Omega$$



Here we have base reflected to emitter

$$I_B \Rightarrow I_E = (\beta+1)I_B$$

$$R_B \Rightarrow \frac{R_B}{\beta+1}$$

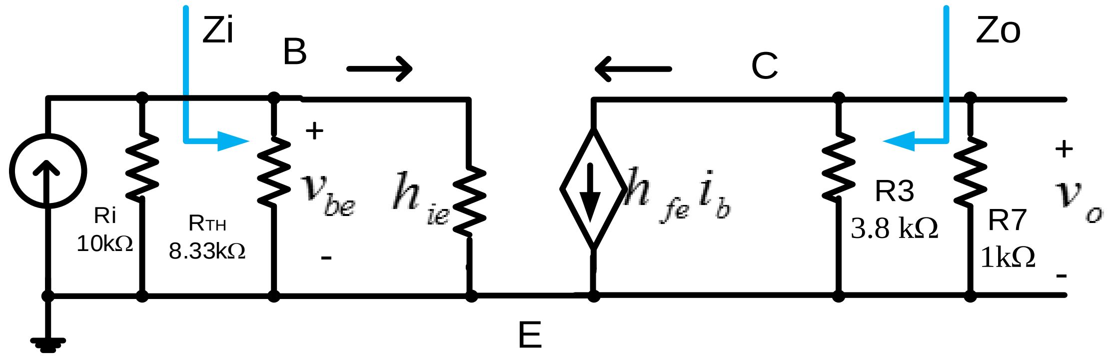


Here we have emitter reflected to base

$$I_E \Rightarrow I_B = \frac{I_E}{(\beta+1)}$$

$$R_E \Rightarrow R_E(\beta+1)$$

AC small signal Equivalent Circuit



1) $$A_V = \frac{v_o}{v_i}$$

$$A_V = \frac{v_o}{v_i} = \frac{v_o}{i_b} \cdot \frac{i_b}{v_i}$$

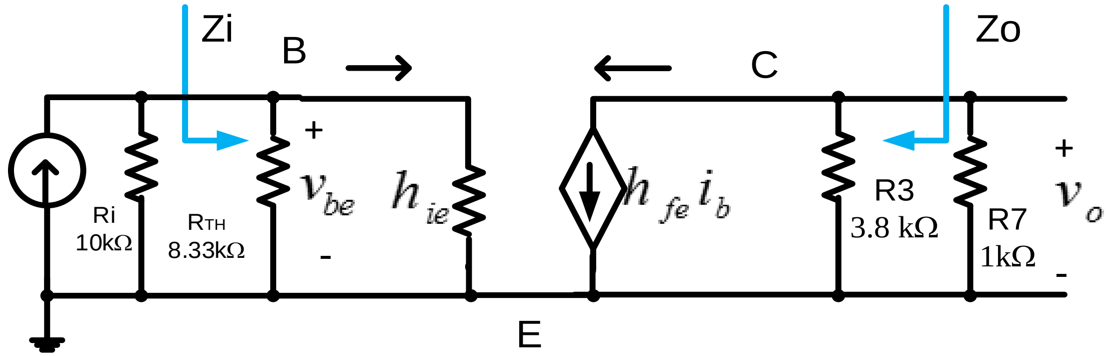
$$v_o = -h_{fe} i_b . (R_3 // R_7)$$

$$i_b = \frac{v_i}{h_{ie}}$$

$$= -h_{fe} . (R_3 // R_7) . \left(\frac{1}{h_{ie}} \right)$$

$$= -50 . (3.8\,\mathrm{k\Omega} // 1\,\mathrm{k\Omega}) . \left(\frac{1}{928\,\Omega} \right) = -42.7$$

AC small signal Equivalent Circuit



$$2)\ Z_I = R_{TH} // h_{ie}$$

$$= 8.33\ \text{k}\Omega // 928\ \Omega$$

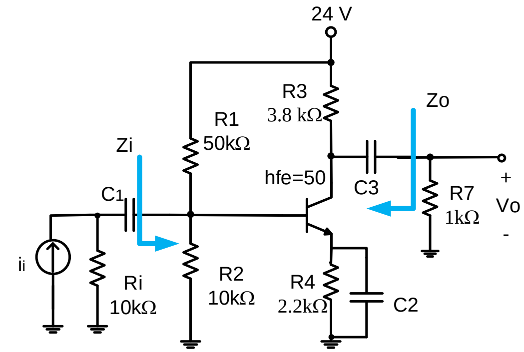
only elements to the right of arrow are considered

according to the given direction of the arrow

$$3)\ Z_o \Big|_{\text{all independant sources killed (i.e. Vi=0 or short)}} = 3.8\ \text{k}\Omega$$

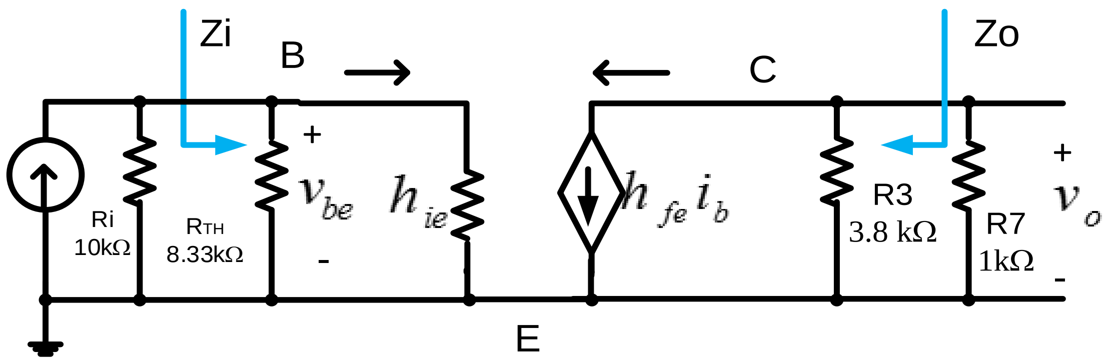
$$\text{here } h_{fe}.i_b = 0 \ \text{ since } i_b = 0 \ \ (vi = 0 \ \text{ - killed})$$

AC small signal Equivalent Circuit



4) $A_i = \dfrac{i_o}{i_i}$

$$i_b = (i_i)\left(\frac{R_I // R_{TH}}{(R_I // R_{TH}) + h_{ie}}\right)$$

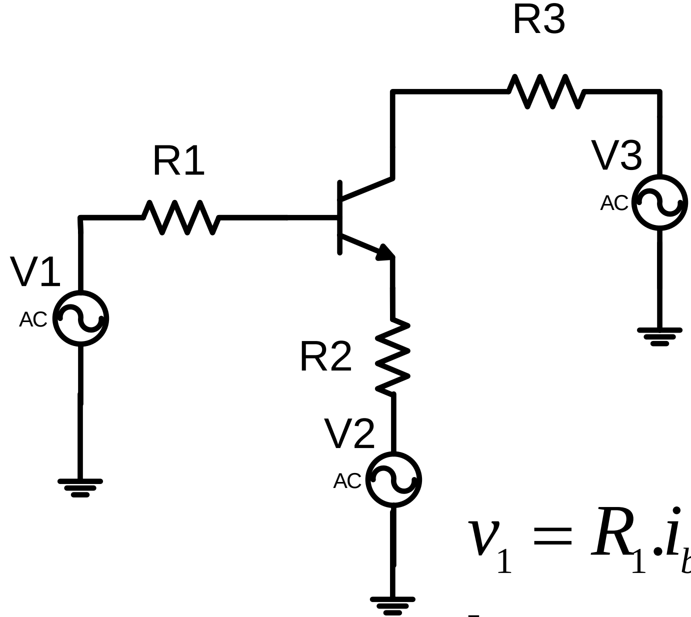


$$i_o = -h_{fe} i_b \left(\frac{R_3}{R_3 + R_7}\right)$$

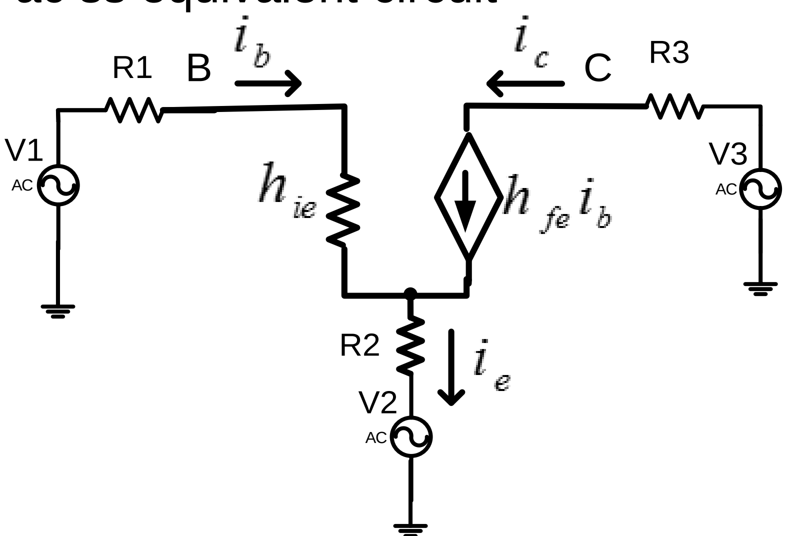
$$A_i = \frac{i_o}{i_i} = \frac{i_o}{i_b} \cdot \frac{i_b}{i_i} = -h_{fe}\left(\frac{R_3}{R_3 + R_7}\right) \cdot \left(\frac{R_I // R_{TH}}{(R_I // R_{TH}) + h_{ie}}\right) = -33$$

Impedance Reflection Concept

V1,V2,V3 are
all ac sources



ac ss equivalent circuit



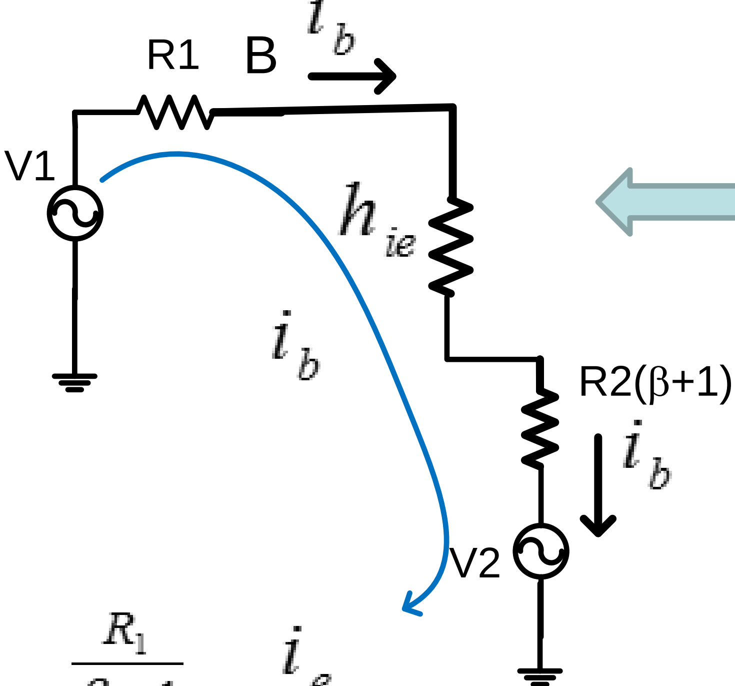
$$v_1 = R_1.i_b + h_{ie}.i_b + R_2.i_e + v_2$$

$$\text{but } i_e = (\beta + 1)i_b$$

$$v_1 = R_1.i_b + h_{ie}.i_b + R_2.(\beta + 1)i_b + v_2$$

$$i_b = \frac{v_1 - v_2}{R_1 + R_2.(\beta + 1)} \Leftarrow \text{base loop}$$

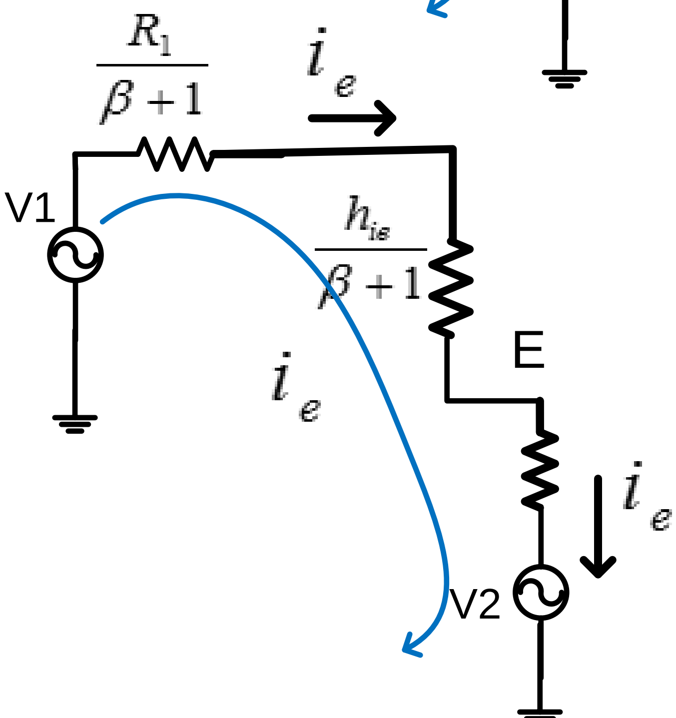
equivalent circuit equation



$$i_b = \frac{v_1 - v_2}{R_1 + h_{ie} + R_2.(\beta + 1)} \Leftarrow \text{base loop}$$

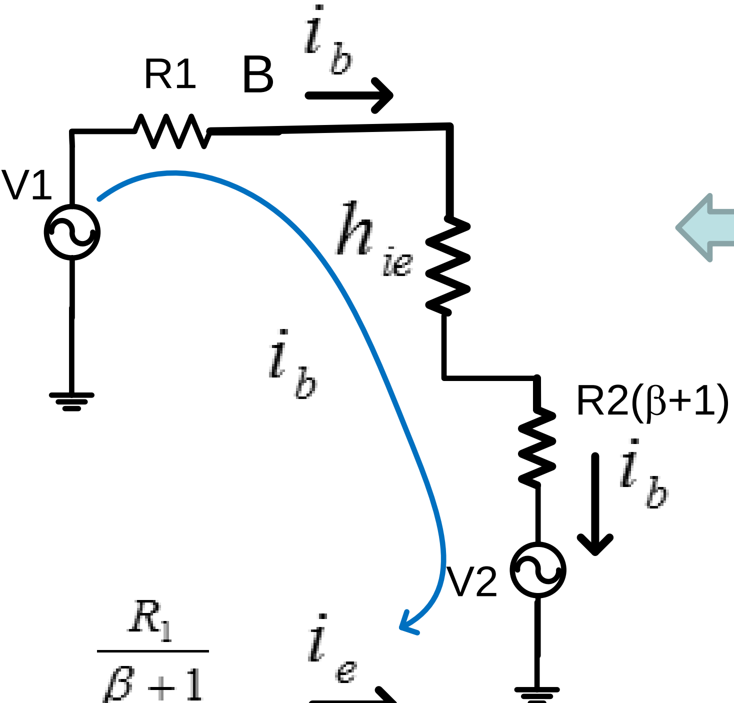
equivalent circuit equation

Instead of i_b use i_e



$$i_e = \frac{v_1 - v_2}{\frac{R_1}{(\beta+1)} + \frac{h_{ie}}{(\beta+1)} + R_2} \Leftarrow \text{Emitter loop}$$

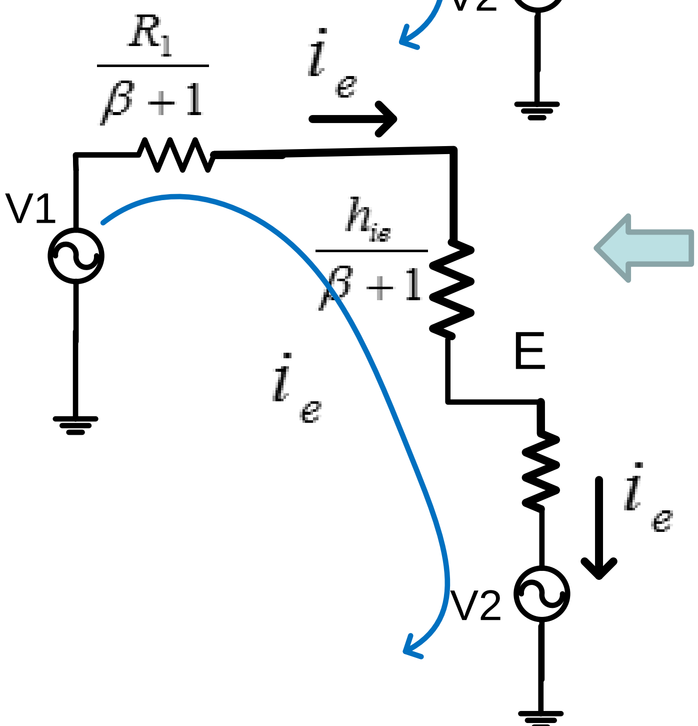
equivalent circuit equation



base equivalent circuit

(reflection from emitter to base)

Here we must change i_e to i_b which requires division by $(h_{fe}+1)$, but voltage must remain the same and thus the resistance must be multiplied by the same factor $(h_{fe}+1)$

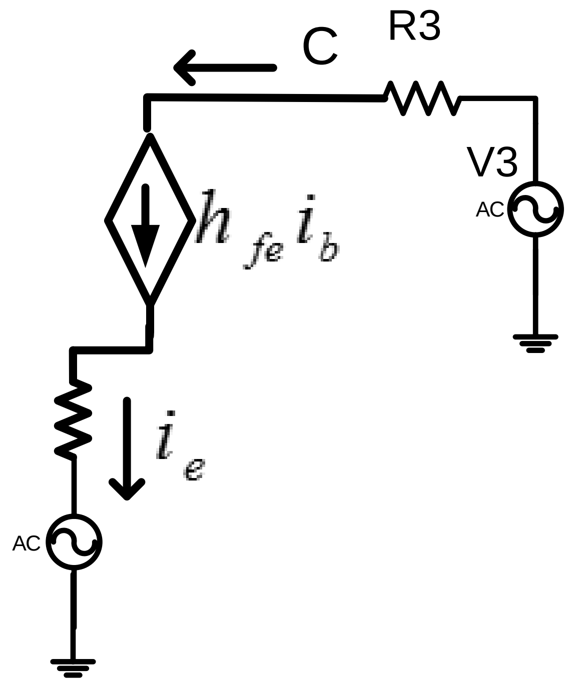


Emitter equivalent circuit

(reflection from base to emitter)

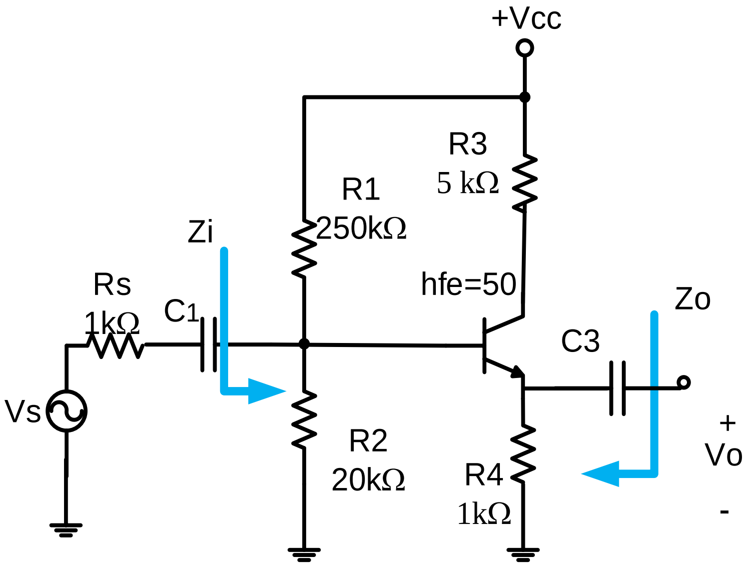
Here we must change i_b to i_e which requires multiplication by $(h_{fe}+1)$, but voltage must remain the same and thus the resistance must be divided by the same factor $(h_{fe}+1)$

Collector Equivalent Circuit



Note: there is no reflection from emitter to collector or vise versa since the i_e and i_c are almost the same

Common Collector Amplifier



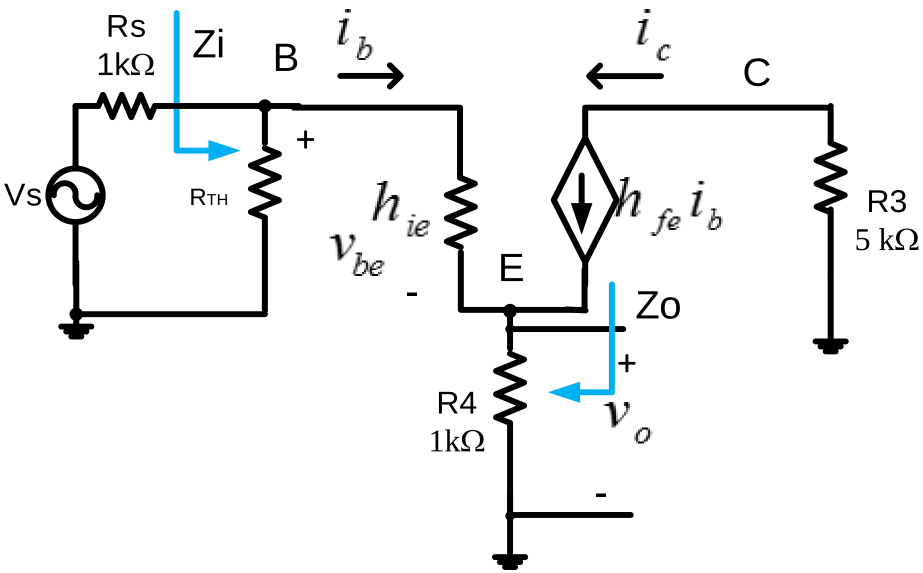
Given

$$h_{ie} = 1k\Omega$$

$$h_{fe} = \beta = 50$$

Find Av, Ai, Zi , Zo

AC small signal Equivalent Circuit

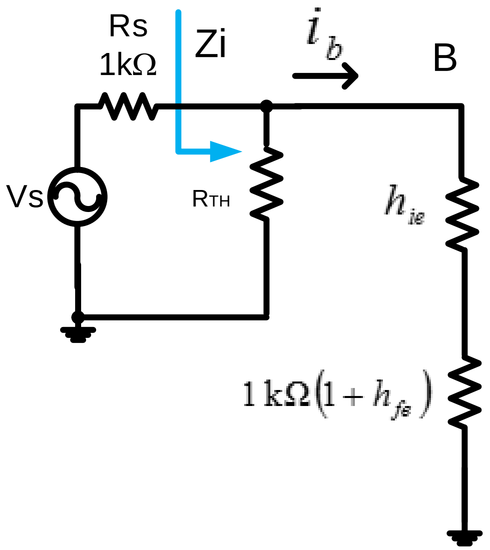


$$1)\ A_V = \frac{v_o}{v_s}$$

$$v_o = 1k\Omega . i_e$$

$$i_e = i_b (h_{fe} + 1)$$

i_b can be found from base equivalent circuit



$$R_{TH} = 20\,k\Omega // 250\,k\Omega$$

$$i_b = i_i \frac{R_{TH}}{(R_{TH}) + (h_{ie} + 1\,k\Omega(h_{fe}+1))}$$

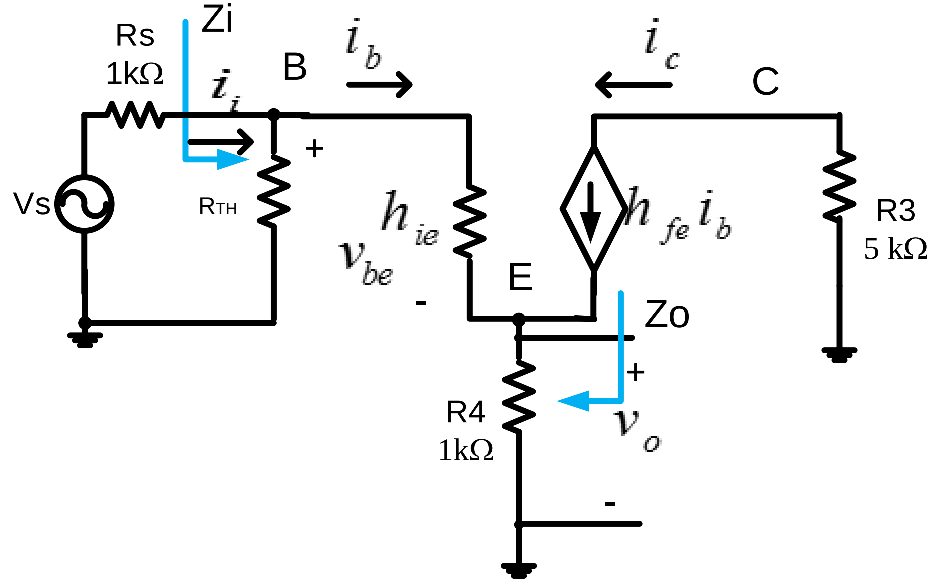
$$i_i = \frac{V_S}{R_S + (R_{TH} // (h_{ie} + 1\,k\Omega(h_{fe}+1)))}$$

$$\therefore \mathrm{A_V} = \frac{v_o}{v_S} = \frac{v_o}{i_e} \cdot \frac{i_e}{i_b} \cdot \frac{i_b}{i_i} \cdot \frac{i_i}{v_S}$$

$$= (1\mathrm{k}\Omega).(\mathrm{h_{fe}} + 1)\left(\frac{R_{TH}}{(R_{TH}) + (h_{ie} + 1\,k\Omega(h_{fe}+1))}\right)\left(\frac{1}{R_S + (R_{TH} // (h_{ie} + 1\,k\Omega(h_{fe}+1)))}\right)$$

$$= 0.915 < 1$$

$$2)A_i = \frac{i_o}{i_i}$$



$$i_o = \frac{v_o}{1\,\text{k}\Omega}$$

$$i_o = i_e = i_b(h_{fe}+1)$$

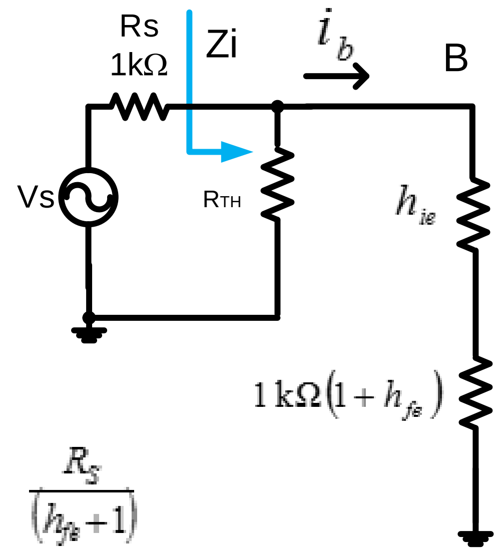
$$i_b = i_i \frac{R_{TH}}{(R_{TH}) + \left(h_{ie} + 1\,\text{k}\Omega(h_{fe}+1)\right)}$$

$$A_i = \frac{i_o}{i_i} = \frac{i_o}{i_e} \cdot \frac{i_e}{i_b} \cdot \frac{i_b}{i_i}$$

$$= 1(h_{fe}+1)\left(\frac{R_{TH}}{R_{TH} + [hie + 1k(hfe+1)]}\right) = 13.39 > 1$$

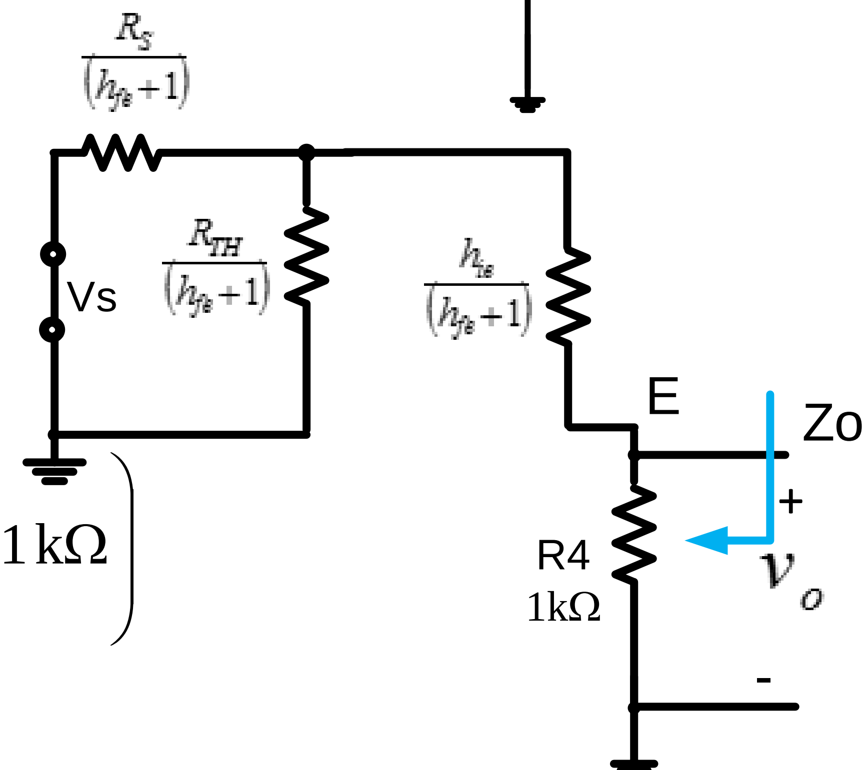
$$3)\ Z_I = \left(R_{TH} // \left(h_{ie} + 1\,\text{k}\Omega(h_{fe}+1)\right)\right)$$

$$= 13.66\,\text{k}\Omega \quad (\text{high})$$



Emitter Equivalent Circuit

$\& V_S = 0$



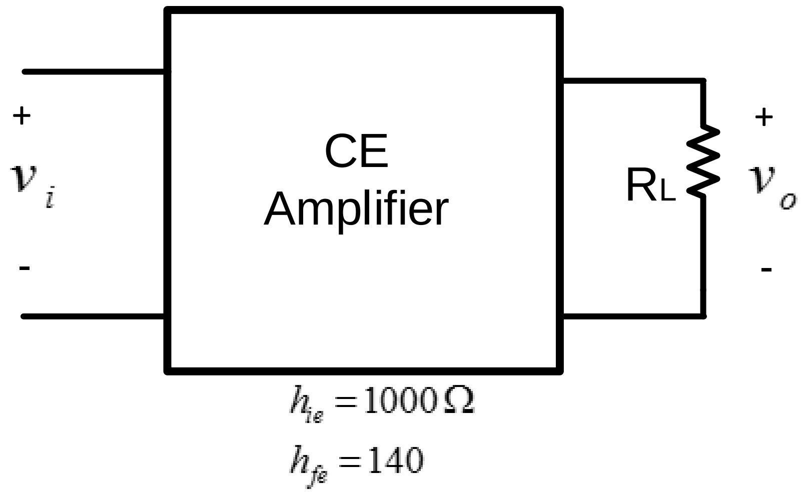
$$Z_o\big|_{Vs=0} = \left(\frac{(R_S // R_{TH}) + h_{ie}}{(h_{fe}+1)} // 1\,\text{k}\Omega\right)$$

$$= \left(\left(\left(\frac{R_S}{(h_{fe}+1)} // \frac{R_{TH}}{(h_{fe}+1)}\right) + \frac{h_{ie}}{(h_{fe}+1)}\right) // 1\,\text{k}\Omega\right)$$

$$= 36.8\,\Omega \quad (\text{low})$$

CC Amplifier as a Buffer

- The value of load resistor RL affects the voltage gain Av,
- This effect is called loading effect and can be substantial



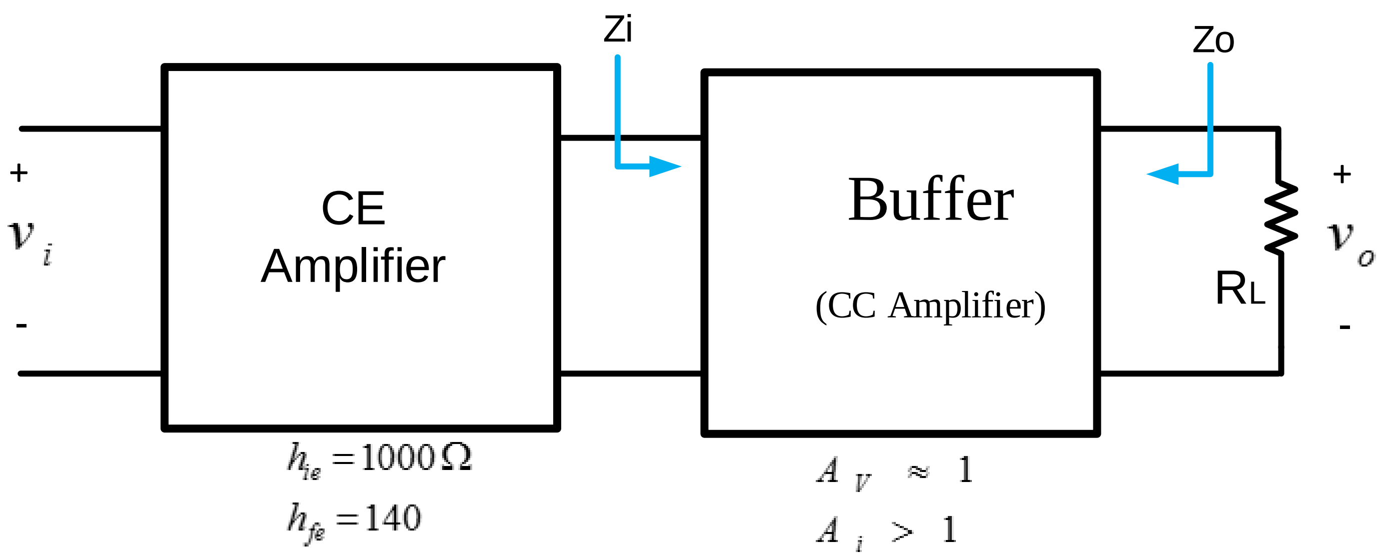
- A buffer (interface) can be used between the amplifier and the load to reduce this loading effect and keep the high gain
- CC Amplifier is also known as Emitter Follower

CC Amplifier as a Buffer

- The buffer must have the following characteristic:

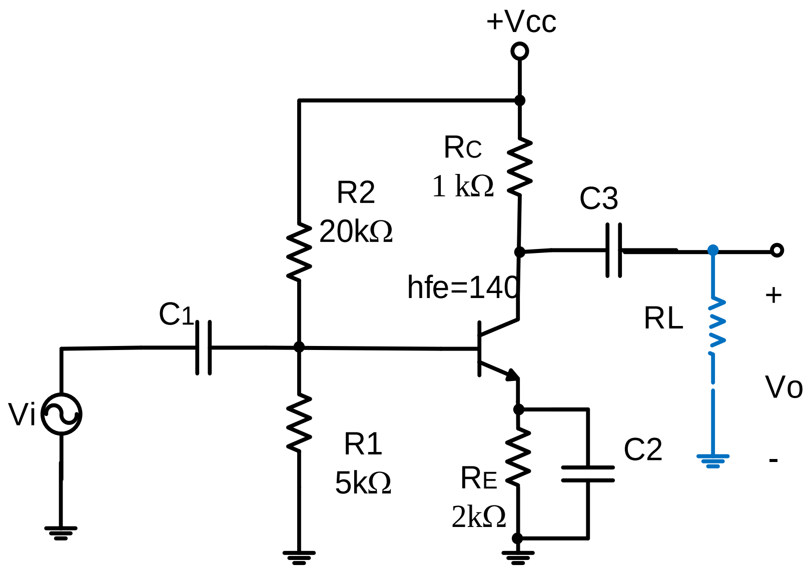
$$A_V \approx 1$$
$$A_I > 1$$
$$Z_I >> \text{high}$$
$$Z_o << \text{low}$$

- The above characteristic are present in the CC amplifier the load to reduce this loading effect and keep the high gain



Example

- First we consider effect of load (RL) on amplifier voltage gain
- Then we use a buffer and see its effect on reducing effect of RL



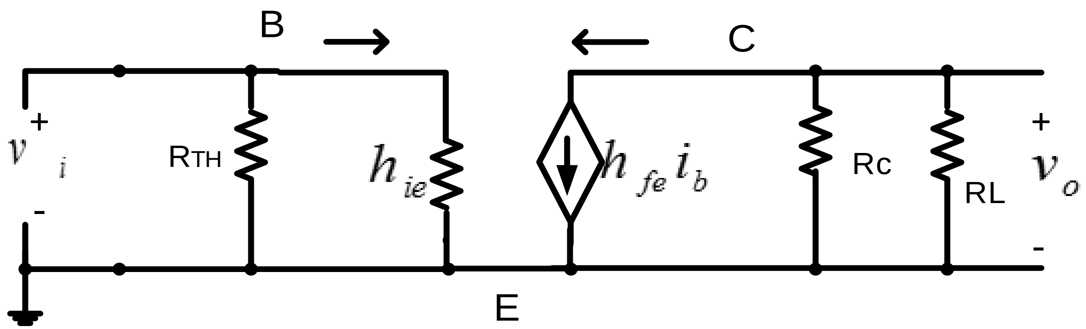
1) with $R_L = \infty$

$$v_o = -h_{fe} i_b . (R_C)$$

$$i_b = \frac{v_i}{h_{ie}}$$

$$A_V = \frac{v_o}{v_i} = \left(-h_{fe} R_C\right) . \frac{1}{h_{ie}} = -140$$

2) with $R_L = 50\,\Omega$

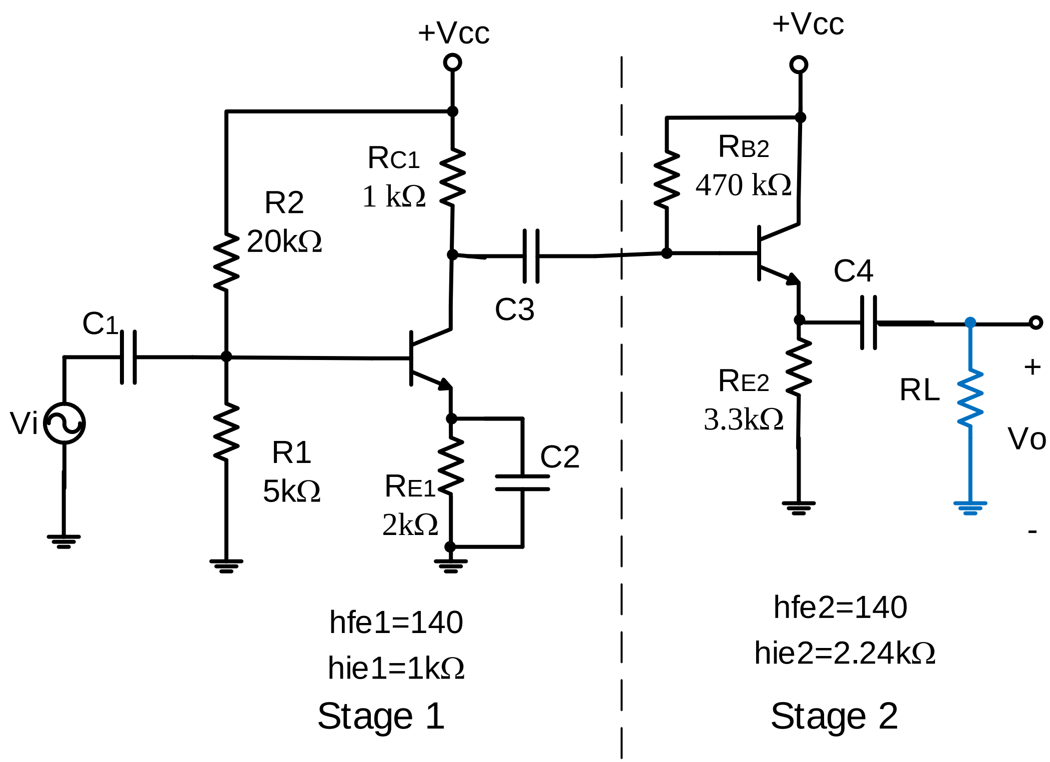


$$A_V = \frac{v_o}{v_i} = \left(-h_{fe} R_C\right) . \frac{1}{h_{ie}} = -6.87$$

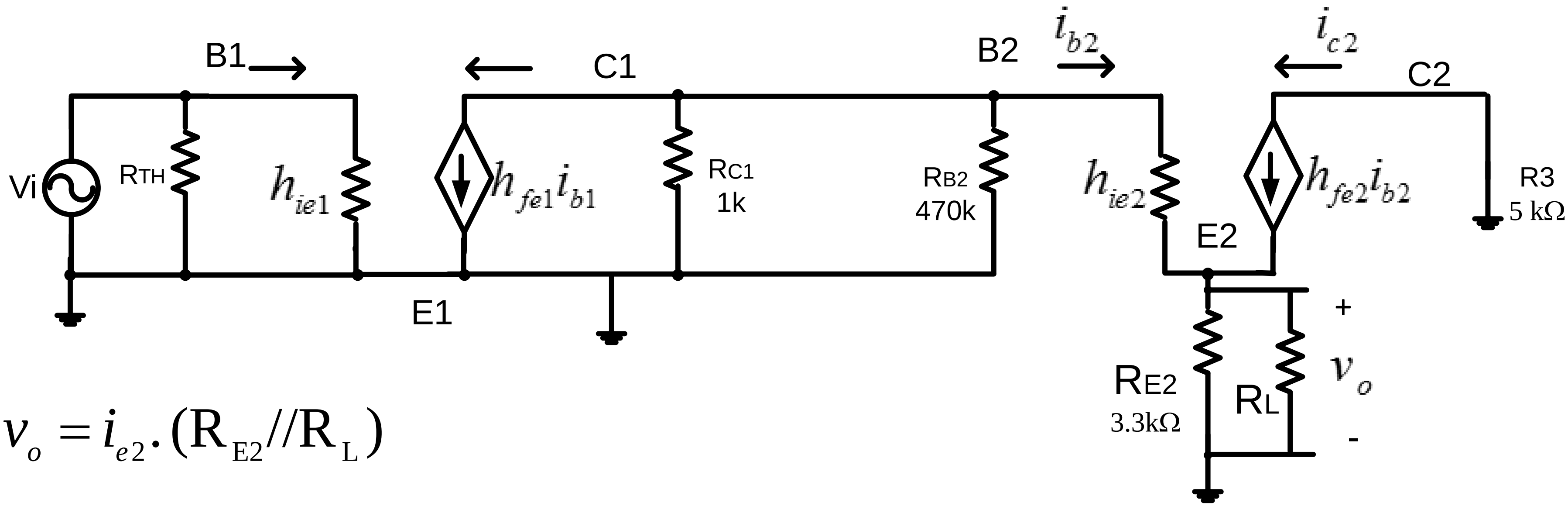
Av have been reduced from -140 to -6.87

Amplifier + Buffer + Load

Now let us look at the new circuit with the buffer



ac ss equivalent Circuit



$$v_o = i_{e2}.(\mathrm{R_{E2}}//\mathrm{R_L})$$

$$i_{e2} = i_{b2}(1+h_{fe2})$$

$$i_{b2} = -h_{fe1}.i_{b1}\frac{(R_{C1}//R_{B2})}{((R_{C1}//R_{B2})+(h_{ie2}+(R_{E2}//R_L)(1+h_{fe2})))}$$

$$i_{b1} = \frac{V_i}{h_{ie1}}$$

$$\Rightarrow Av = \frac{v_o}{v_i} = \frac{v_o}{i_{e2}}.\frac{i_{e2}}{i_{b2}}.\frac{i_{b2}}{i_{b1}}.\frac{i_{b1}}{v_i} = -95.6$$

This is much better than the case without buffer

Multistage Amplifiers

- The previous example of a CE amplifier with a CC buffer is an example of a multistage amplifier (two-stage amplifier)
- Multistage amplifiers can be used to get more gain and to improve the performance of the amplifier
- These amplifiers such that the Output of first stage is connected to input of second stage
- Capacitor C3 is a decoupling capacitor that separates the two stages for DC bias point stability, this makes the two stages completely separate in DC analysis and their Q-points are not affected by each other
- C2 is used as a bypass capacitor for stage 1 and allows stabilization of the Q-point, if C2 is removed the input impedance of the amplifier can be improved

Cascaded Systems

- The output of one amplifier is the input to the next amplifier
- The overall voltage gain is determined by the product of gains of the individual stages
- The DC bias circuits are isolated from each other by the coupling capacitors
- The DC calculations are independent of the cascading
- The AC calculations for gain and impedance are interdependent

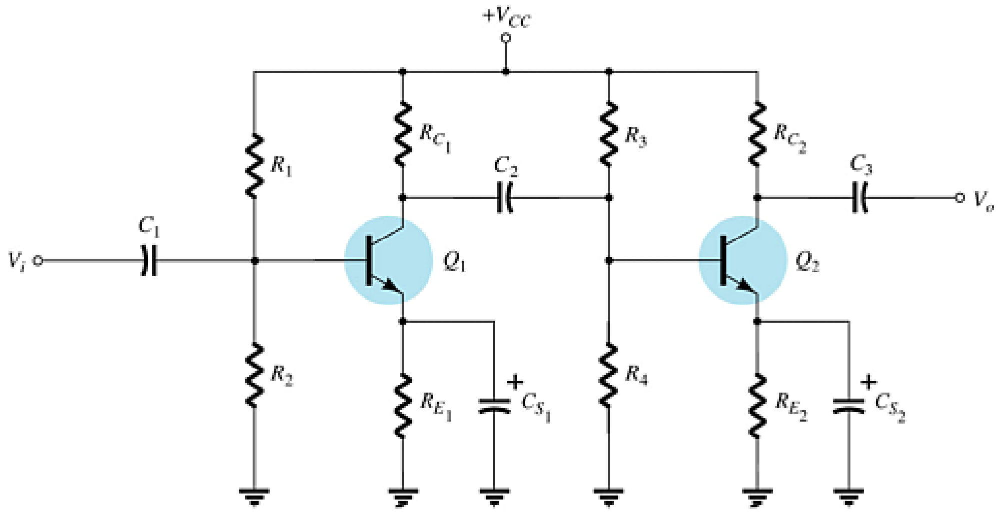
R-C Coupled BJT Amplifiers

Voltage gain:

$$A_v = A_{v1} A_{v2}$$

Input impedance, first stage:

$$Z_i = R_1 \parallel R_2 \parallel h_{ie1}$$

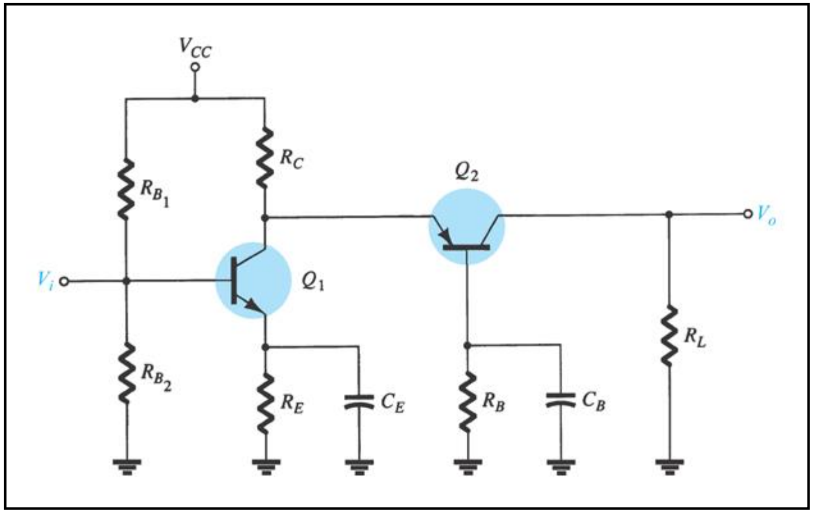


Output impedance, second stage:

$$Z_o = R_C$$

Cascode Connection

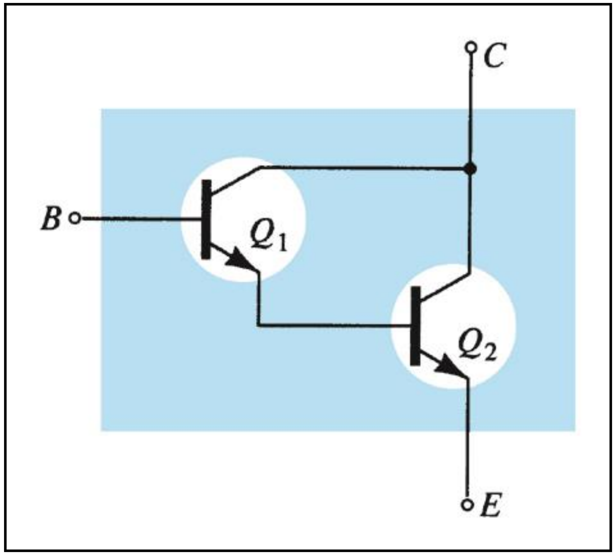
- This example is a CE–CB combination. This arrangement provides high input impedance but a low voltage gain.
- The low voltage gain of the input stage reduces the Miller input capacitance, making this combination suitable for high-frequency applications.



Exercise : Find Av, Zi and Zo

Darlington Connection

- The Darlington circuit provides very high current gain, equal to the product of the individual current gains:
  - $\beta_D = \beta_1 \beta_2$
- The practical significance is that the circuit provides a very high input impedance.



DC Bias of Darlington Circuits

Base current: $$I_{BD} = I_{B1} = \frac{V_{CC} - V_{BED}}{R_B + (\beta_D + 1)R_E}$$

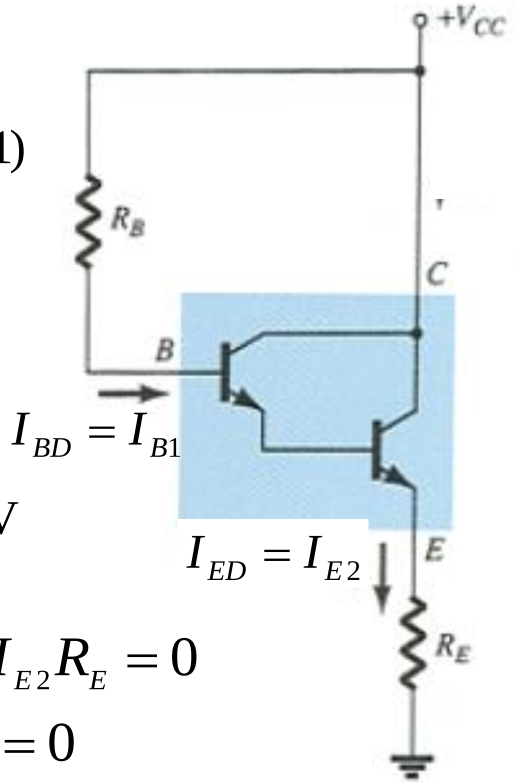
Emitter current:

$$I_{ED} = I_{E2}$$
$$I_{E2} = I_{B2}(\beta_2 + 1)$$
$$I_{B2} = I_{E1}$$
$$I_{E1} = I_{B1}(\beta_1 + 1)$$
$$I_{E2} = I_{B1}(\beta_2 + 1)(\beta_1 + 1)$$
$$I_{ED} = \beta_D I_{BD}$$

Emitter voltage: $$V_E = I_{ED} R_E$$

Base voltage:

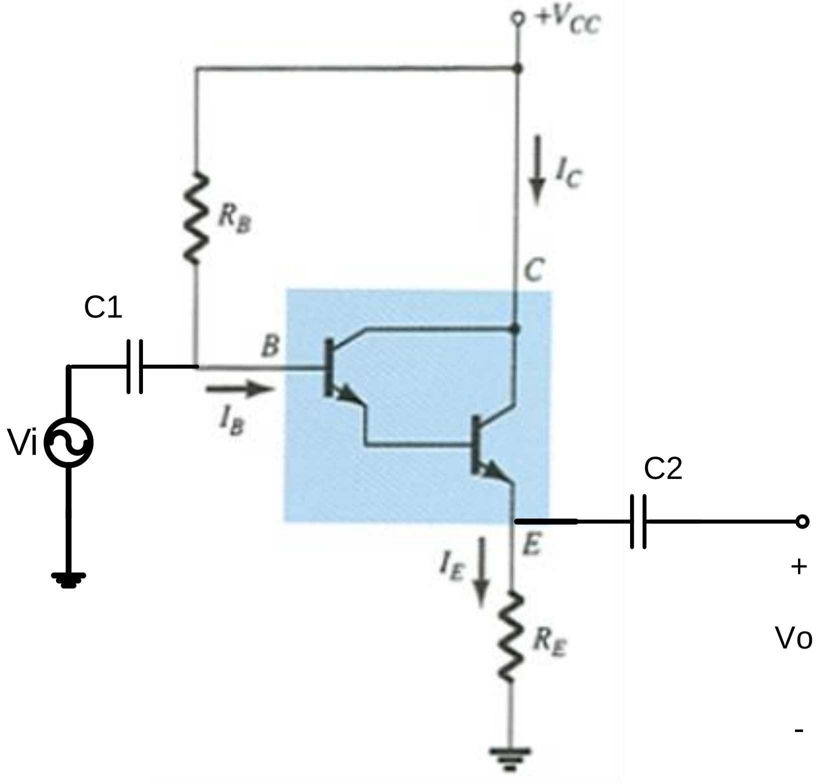
$$V_B = V_E + V_{BE}$$
$$V_{BED} = V_{BE1} + V_{BE2} \cong 1.4\ \text{V}$$

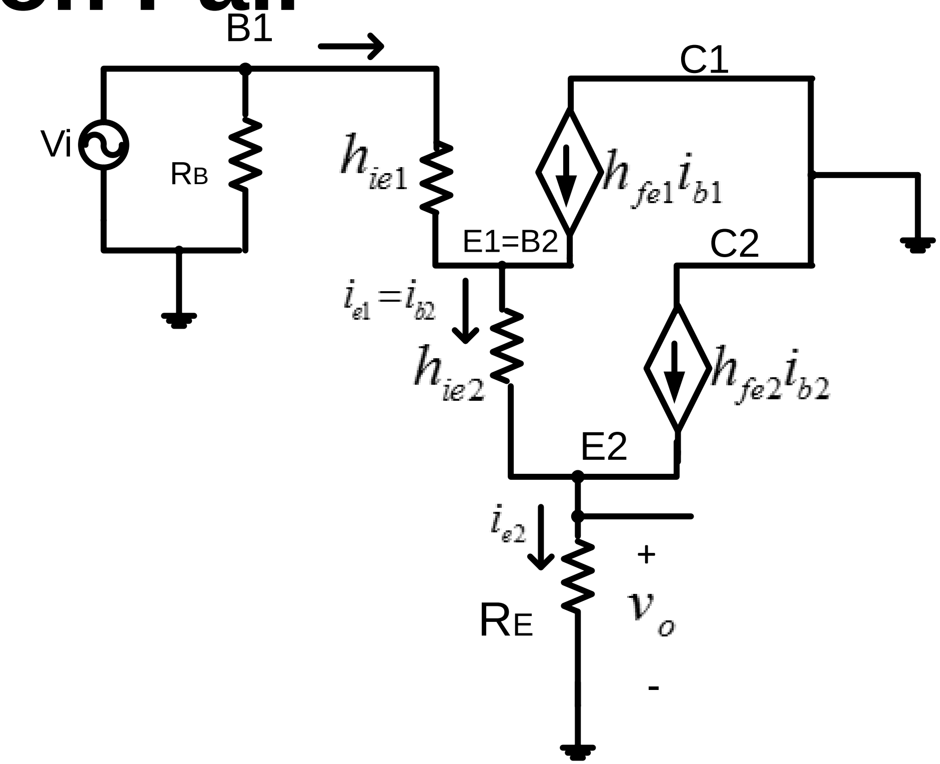


KVL for input loop :

$$V_{CC} - I_{B1}R_B - V_{BE1} - V_{BE2} - I_{E2}R_E = 0$$
$$V_{CC} - I_{BD}R_B - V_{BED} - I_{ED}R_E = 0$$

Darlington Pair





Find Ratio of $\frac{\mathrm{i_{e2}}}{\mathrm{i_{b1}}}$ and $\mathrm{A_v}$

$$i_{e2} = i_{b2}(h_{fe2}+1)$$

$$i_{b2} = i_{e1}$$

$$i_{e1} = i_{b1}(h_{fe1}+1)$$

$$i_{e2} = i_{b1}(h_{fe1}+1)(h_{fe2}+1)$$

$$i_{ed} = h_{fed} i_{bd}$$

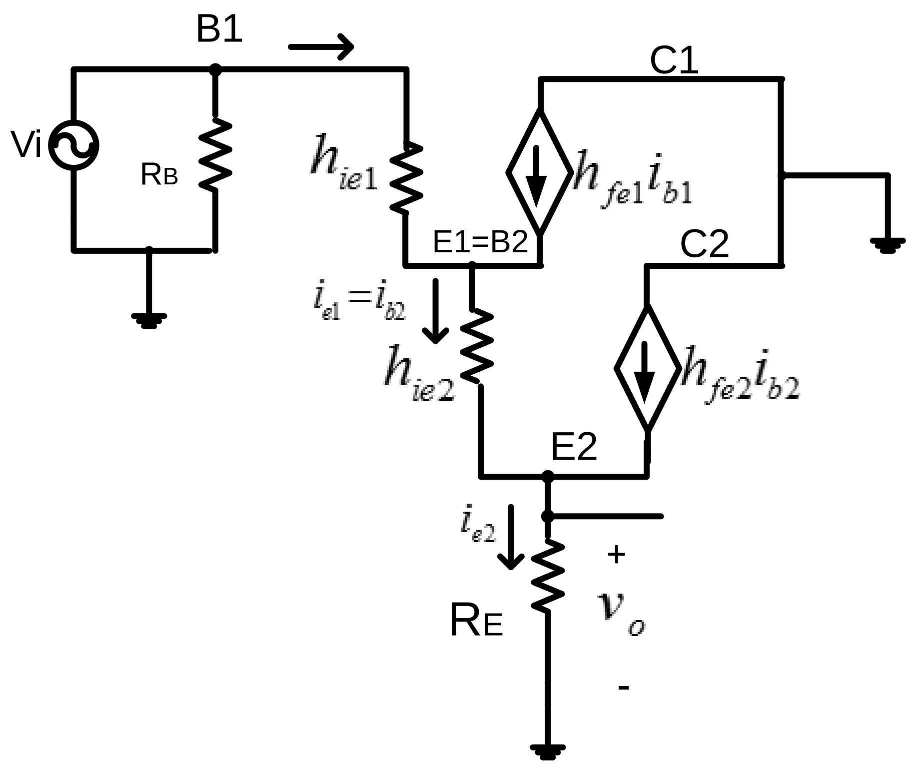
$$\begin{aligned} h_{fed} &= (h_{fe1}+1)(h_{fe2}+1) \\ &\cong h_{fe1}h_{fe2} \\ &\cong h_{fe}{}^{2} \quad , (\text{if } h_{fe1} = h_{fe2} = h_{fe}) \end{aligned}$$

2) Find $A_v = \dfrac{v_o}{v_i}$

$$v_o = i_{e2} R_E$$

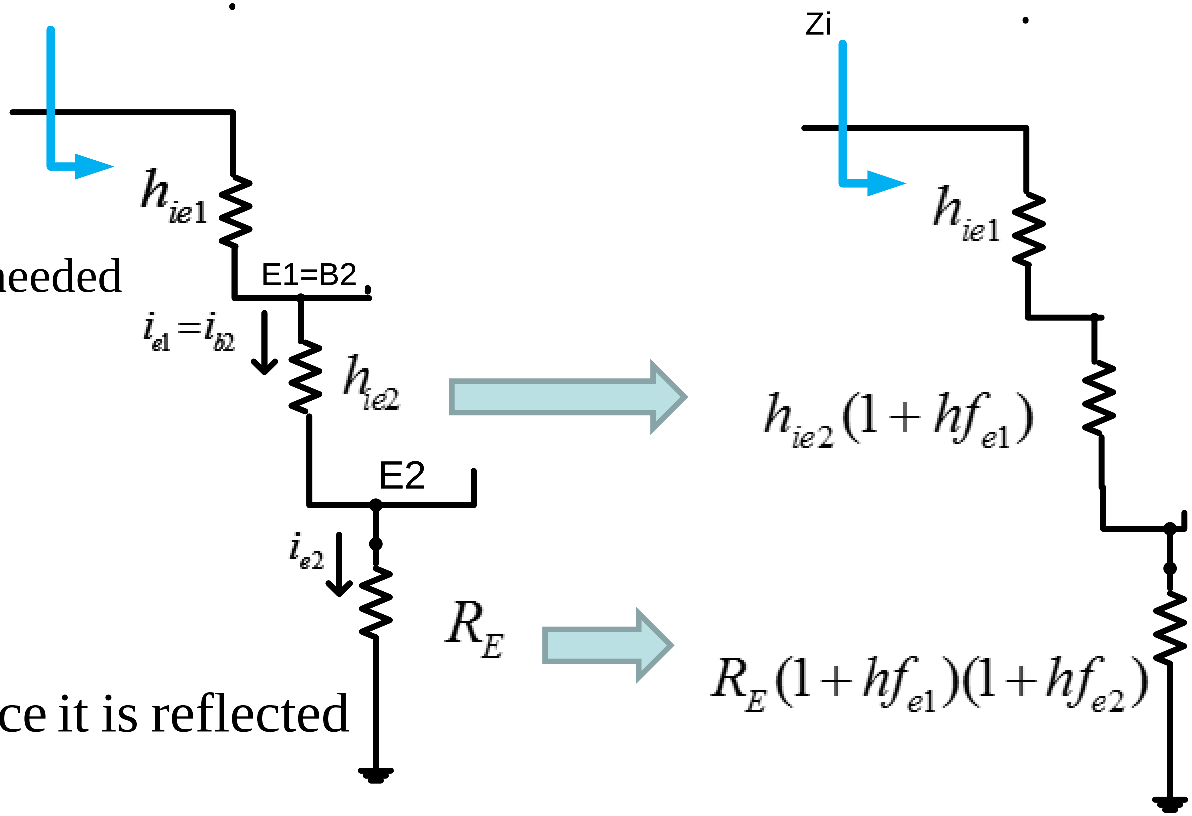
$$i_{e2} = i_{b1}(h_{fe1} + 1)(h_{fe2} + 1)$$

$$i_{b1} = \frac{v_i}{Z_i}$$



3) Find Z_I

base equivalent circuit is needed



$h_{ie2} \Rightarrow h_{ie2}((h_{fe1}+1)$ since it is reflected

from emitter1 to base1

$R_E \Rightarrow R_E(h_{fe1}+1)(h_{fe2}+1)$

since it is reflected twice

1) From E2 to B2 (B2 = E1)

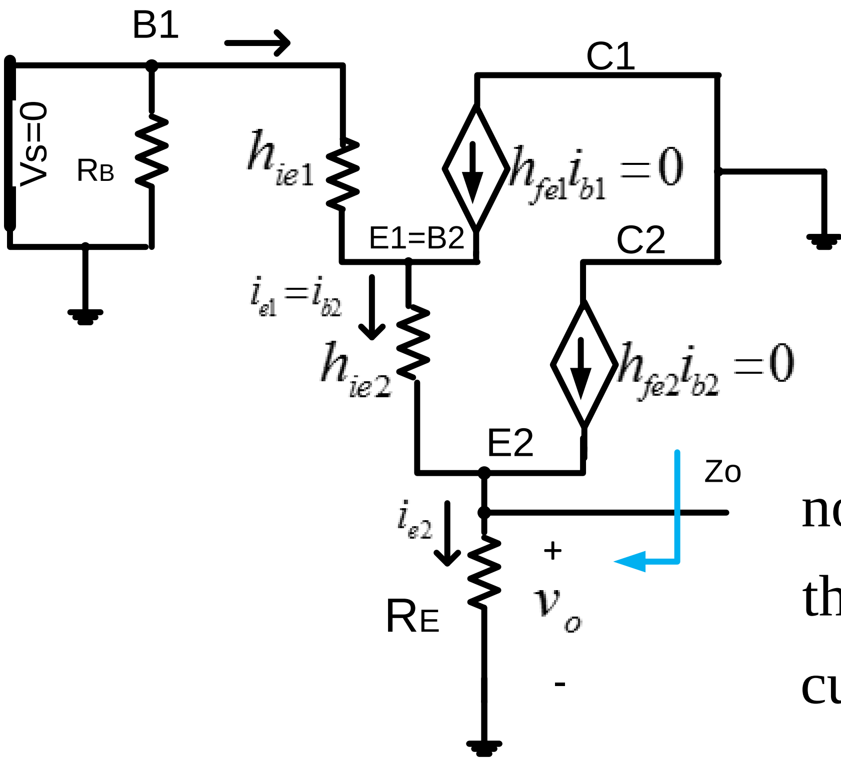
2) From E1 to B1

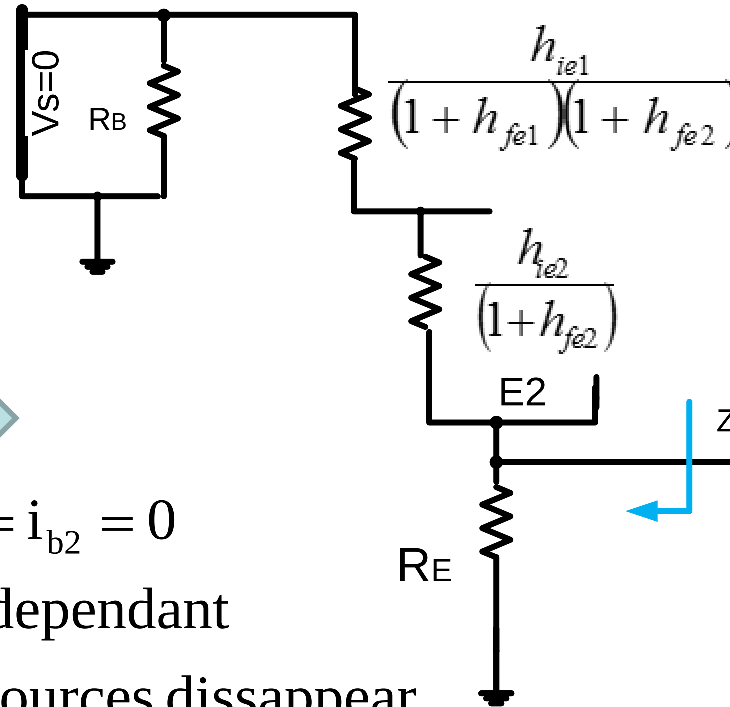
$$Z_I = h_{ie1} + h_{ie2}(h_{fe1}+1) + R_E(h_{fe1}+1)(h_{fe2}+1)$$

Zo

4) Find $Z_o\big|_{Vs=0}$

Emitter equivalent circuit is needed





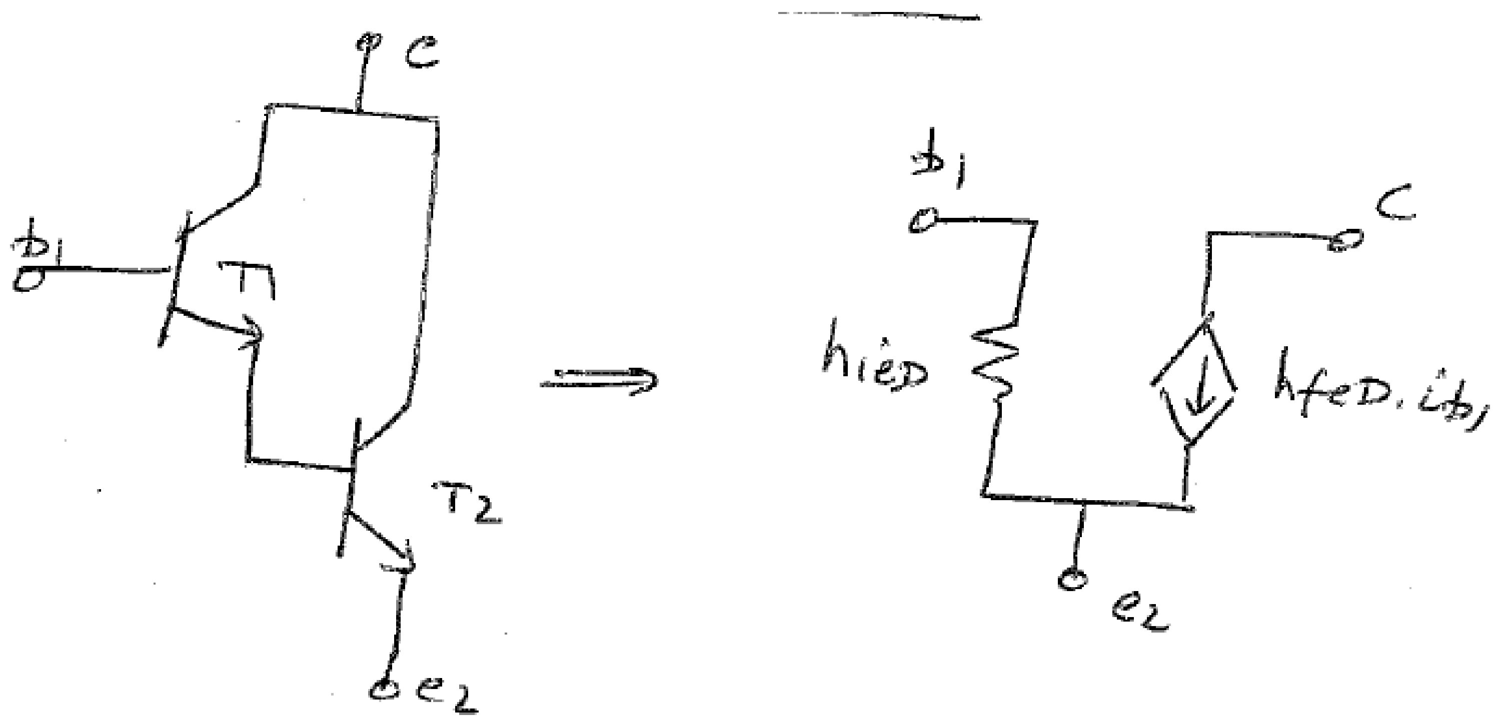
note $i_{b1} = i_{b2} = 0$

the two dependant

current sources dissappear

$$Z_0 = \left(\frac{h_{ie1}}{(h_{fe1}+1)(h_{fe2}+1)} + \frac{h_{ie2}}{(h_{fe2}+1)} \right) // R_E$$

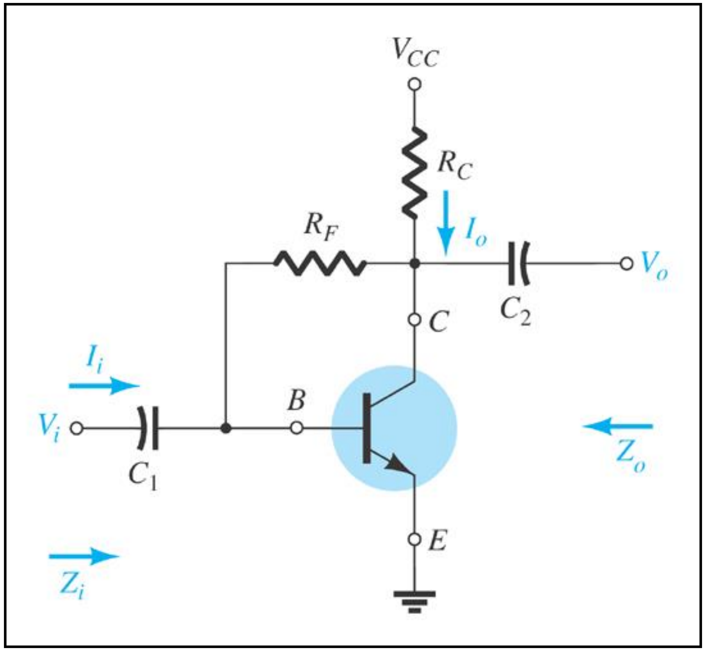
Darlington Simplified Model



$$h_{ieD} \cong 2h_{ie}$$

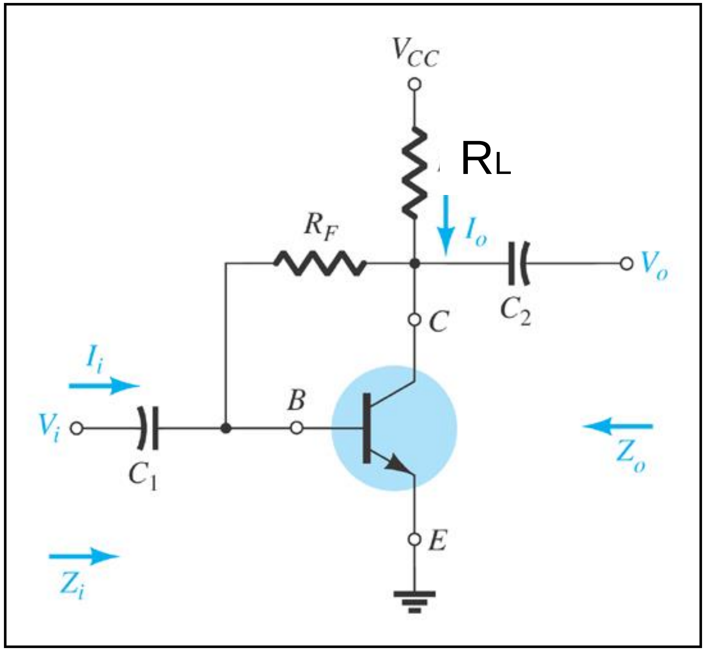
$$h_{feD} \cong h_{fe1}.h_{fe2}$$

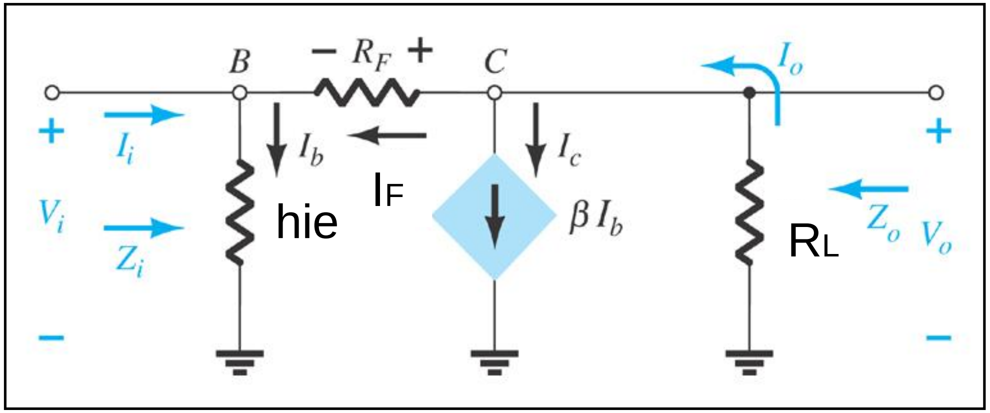
Base To Collector Feedback



Exercise : Find A_V, Zi and Zo

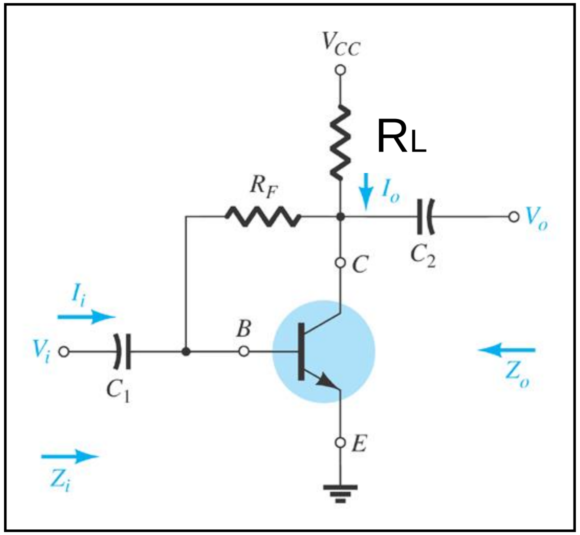
Base To Collector Feedback

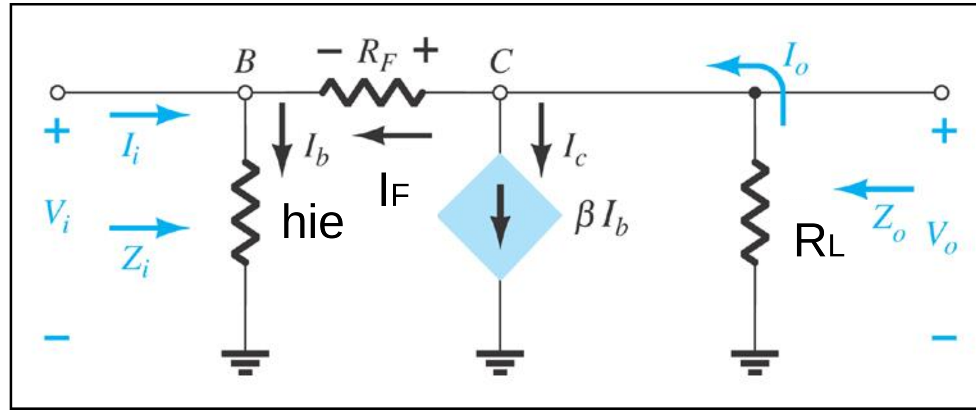




Exercise : Find A_V, Zi and Zo

Base To Collector Feedback





$$v_0 = -i_o.R_L$$

$$i_o = h_{fe}.i_b + i_F$$

$$i_F = \frac{v_o - v_i}{R_F}$$

$$i_b = \frac{v_i}{h_{ie}}$$

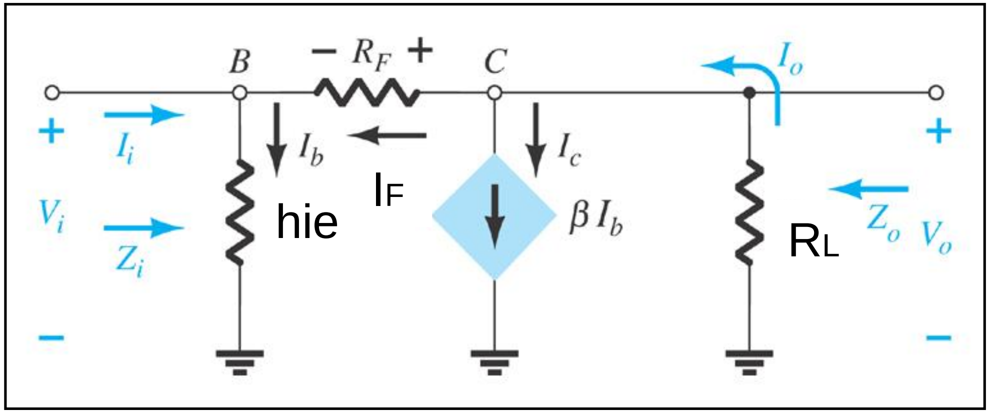
$$v_0 = -\left(h_{fe}.\frac{v_i}{h_{ie}} + \frac{v_o - v_i}{R_F}\right).R_L$$

$$v_0 = -R_L h_{fe}.\frac{v_i}{h_{ie}} - \frac{v_o R_L}{R_F} + \frac{v_i R_L}{R_F}$$

$$v_0\left(1 + \frac{R_L}{R_F}\right) = v_i\left(\frac{R_L}{R_F} - R_L.\frac{h_{fe}}{h_{ie}}\right)$$

$$Av = \frac{\left(\frac{R_L}{R_F} - R_L.\frac{h_{fe}}{h_{ie}}\right)}{\left(1 + \frac{R_L}{R_F}\right)}$$

$$Z_0\Big|_{v_i=0} = R_F \,//\, R_L$$



$$Z_i = \frac{v_i}{i_i}$$

$$i_i = i_b - i_F = \left(\frac{v_i}{h_{ie}} - \frac{v_o - v_i}{R_F}\right)$$

$$Z_i = \frac{v_i}{i_i} = \frac{v_i}{\left(\dfrac{v_i}{h_{ie}} - \dfrac{v_o - v_i}{R_F}\right)}$$

$$= \frac{v_i}{\left(\dfrac{R_F v_i - h_{ie}(v_o - v_i)}{R_F h_{ie}}\right)}$$

$$= \frac{v_i R_F h_{ie}}{\left(R_F v_i - h_{ie}(v_o - v_i)\right)}$$

$$= \frac{v_i R_F h_{ie}}{\left((R_F + h_{ie})v_i - h_{ie} v_o\right)}$$

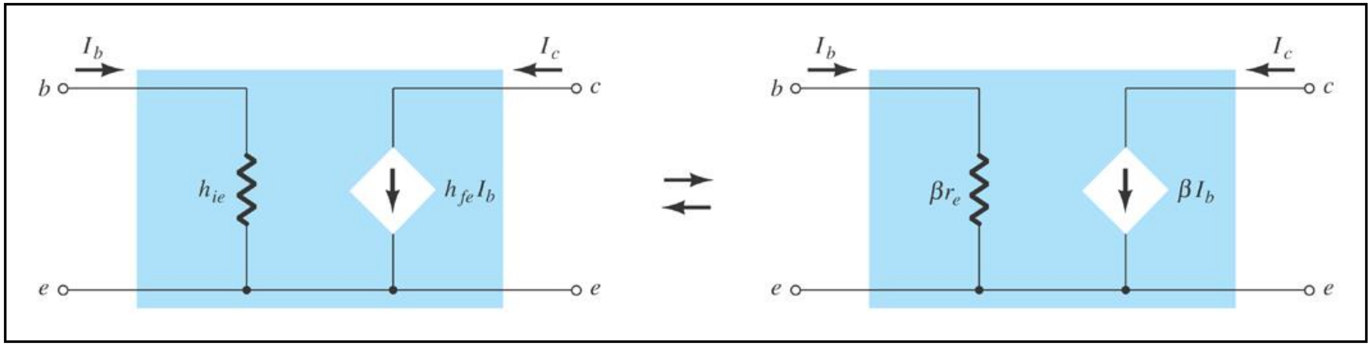
$$= \frac{R_F h_{ie}}{\left((R_F + h_{ie}) - h_{ie}\dfrac{v_o}{v_i}\right)}$$

$$= \frac{R_F h_{ie}}{\left((R_F + h_{ie}) - h_{ie} A_v\right)}$$

r_e vs. h-Parameter Model

Common-Emitter

$$h_{ie} = \beta r_e$$
$$h_{fe} = \beta_{ac}$$



Common-Base

$$h_{ib} = r_e$$
$$h_{fb} = -\alpha \cong -1$$